Making Sense of Financial Mathematics

There are a number of references to financial mathematics in the Leaving Certificate mathematics syllabus

LCFL Strand 3: *solve problems involving*
· finding compound interest
· finding depreciation (reducing balance method)
LCOL Strand 3: *perform calculations involving formulae for compound interest and depreciation (reducing balance method)*
LCHL Strand 3: *use present value when solving problems involving loan repayments and investments*

Perhaps the most insightful learning outcome of all is the following which appears at LCHL Strand 3:

solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment

Yes, you've got it, financial mathematics of this type is simply an application of geometric patterns.

Consider the concept of ***Compound Interest*** by imagining that you have just found out the following startling news: on the day you were born your godfather, Uncle Nicholas, deposited €5000 in your name in a **trust fund** that pays 6% **APR.**
One of the provisions of the trust fund was that you couldn't touch the money until you turned 18. You are now 18 years 10 months old and you are wondering

- How much money is in the trust fund now?
- How much money will be in the trust fund if you wait until your next birthday to cash it in?
- How much money would be in the trust fund if you left the money there until you retire at age 60?

Represent the money that is in your account in a table starting with the day you were born

Time		Amount of money
Day you were born	0	
1st Birthday	1	
2nd Birthday	2	
3rd Birthday	3	
4th Birthday	4	
5th Birthday	5	
6th Birthday	6	

Can you see a pattern in your completed table?

Does this pattern represent an arithmetic sequence or a geometric one? How do you know?

Can you gerneralise the pattern so that you can find out how much will be in the trust fund on your 60th birthday without extending the table?

Look at the tables A and B below

A

Time		Amount of Mon
Day you were born	0	5000
1st Bday	1	5300
2nd Bday	2	5618
3rd Bday	3	5955.08
4th Bday	4	6312.38

B

Time		Amount of Money
Day you were born	0	5000
1st Bday	1	1.06 (5000)
2nd Bday	2	1.06 (1.06)(5000)
3rd Bday	3	1.06(1.06)(1.06)(5000)
4th Bday	4	1.06(1.06)(1.06)(1.06)(5000)

What are the differences between the two tables? What are the similarities between them?

Which table makes it easier to generalise the pattern? Why do you think this is the case?

Generalise the pattern. Is it the same for both tables? How do you know?

In the mathematical tables on page 30 there is a formula for calculating compound Interest.

It reads

$$F = P(1+i)^{t}$$

Where F = the final value

P = the Principal

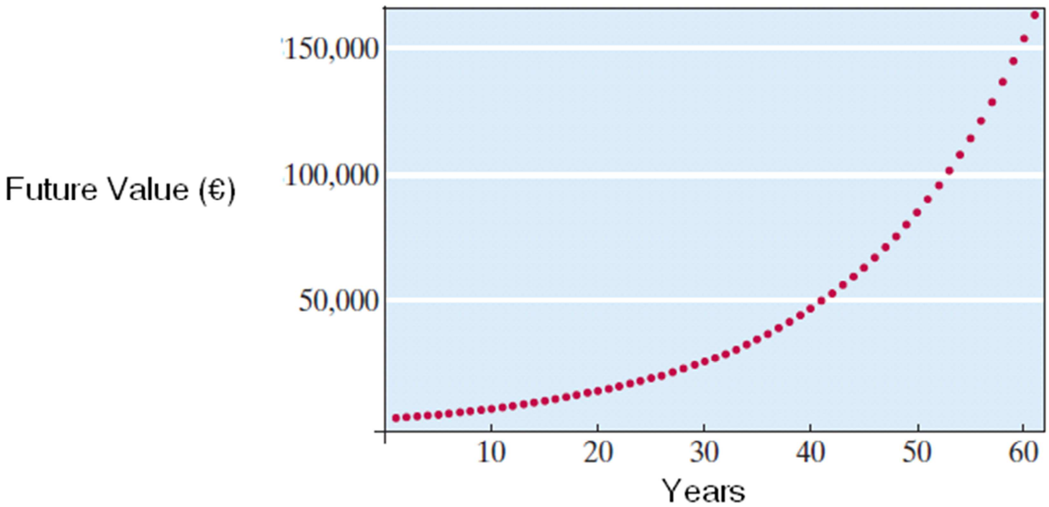
i = the interest rate (annual, or for the relevant period)

t = the time (in years, or other equal intervals of time)

Compare this formula with the generalised expression for the pattern of money in the trust fund.

Are they similar? Can you see the 'F' in your table, ?in your generalised formula? What about the 'P', the 'i', and the 't'?

Below is a graph showing the future value of the trust fund. Are you surprised at the shape of the graph? Why, or why not? Can you find each variable on the graph?



Note that the process of calculating the value of a depreciating asset (reducing balance method) is similar to the process of calculating the value of an interest gaining fixed deposit account, except that the formula has a changed sign:

$$F = P(1 - i)^{t}$$

Can you make out two tables, similar to tables A and B above, showing the value of an asset which cost €5000 if it depreciates each year at a rate of 10%? What is the pattern in this case? Which table makes it easier to generalise the pattern?

Comment on the statement

The future value of an investment at any time t is the n^{th} term of a geometric sequence where P is the initial value of the investment, (1 + i /100) is the common ratio and n is the number of years.

Think …what if Uncle Nicholas had put money into a bank account regularly for you. How could you work out how much it would be worth after a period of time?

Suppose at the start of each year he deposited €500 in an account that pays 3% interest. What will the final amount be after 18 years?

Think about the first €500 he puts in; how many years will it be there for?

The second €500? The third €500 ? for how many years will they be in the account?

How much interest will the first €500 earn? The second €500? The last €500 ?

Ist €500 is in the account for 18 years so will be worth **F = 500(1+0.03)^{18} €851.22**

2nd €500 is in the account for 17 years so will be worth **F = 500(1+0.03)^{17} €826.42**

3rd €500 is in the account for 16 years so will be worth **F = 500(1+0.03)^{16} €802.35**

Can you see the pattern? Is it increasing or decreasing? Why? What will the smallest number be? Why?

So, to calculate how much is in there after 18 years add them all up…or, yes, get the sum of the sequence.

851.22+826.42+803.35+……………………………………………………..+515 =

The tables give you the formula $S_n = \dfrac{a(r^n - 1)}{r - 1}$

What is 'n' in this situation? What is 'a'? what is 'r'?

Now can you answer the question: How much would the money Uncle Nicholas put in the account each year amount to after 18 years?

You might be wondering whether you would have more money if the bank added the interest every month? Try it out and see.

It's really the same situation, except now you have to think about what the interest rate would be if for example the interest was compounded every month instead of every year.

Again it's the same idea: $(1+ i)^{12}$ is now going to be the same as 1.03…provided the 'i' is the monthly rate Why is this? What will be the value of 'n' if a monthly rate is used?

Investigate how much money would be in the account if Uncle Nicholas's bank had compounded the interest monthly.

Now, how would this change if they compounded it daily? hourly? How would the relationship $(1+i)^{12} = 1.03$ change? How would the number of terms in the sequence change?

How would things change if the interest was added at the beginning of the month as opposed to the end of the month?

These are all things that you must consider when answering questions about investments and loans. Sometimes, if the question is not very specific, you may have to make assumptions; in other cases, the question may be very specific and you won't need to make any assumptions. Where you do make assumptions, it is advisable to make these clear.

Now, work through the financial maths questions provided. Check the solutions to see if you are working correctly, or to get an insight into the thinking involved (by clicking on the thought bubble).