

Mathematics in Early Childhood and Primary Education (3-8 years)

Teaching and Learning

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Acronyms

AAMT	Australian Association of Mathematics Teachers
Aistear	The Early Childhood Curriculum Framework (2009)
ACARA	Australian Curriculum, Assessment and Reporting Authority
AfA	Achievement for All (UK intervention)
ASD	Autistic Spectrum Disorders
CCEA	Council for Curriculum, Examinations and Assessment (Northern Ireland)
CCK	Common Content Knowledge
CCSSM	Common Core States Standards for Mathematics (United States)
COMET	Cases of Mathematics Instruction to Enhance Teaching (Silver et al., 2007)
CPD	Continuing Professional Development
DEIS	Delivering Equality of Opportunities in Schools
DES	Department of Education and Skills (formerly Department of Education and Science)
ECA	Early Childhood Australia
ECCE	Early Childhood Care and Education
EMS	European Mathematical Society
HSCL	Home School Community Liaison (Ireland)
ICT	Information and Communications Technology
IWB	Interactive White Board
KCC	Knowledge of Content and Curriculum
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
KQ	Knowledge Quartet (Rowland, Huckstep, & Thwaites, 2005)
MKT	Mathematical Knowledge for Teaching
NAEYC	National Association for the Education of Young Children (United States)
NCCA	National Council for Curriculum and Assessment
NCTM	National Council of Teachers of Mathematics (United States)
NRC	National Research Council (United States)
PCK	Pedagogical Content Knowledge
PSMC	Primary School Mathematics Curriculum (1999)
PUFM	Profound Understanding of Fundamental Mathematics (Ma, 1999)
REPEY	Researching Effective Pedagogy in the Early Years (Siraj-Blatchford et al., 2002)
RME	Realistic Mathematics Education
SCK	Specialised Content Knowledge
SEND	Special Educational Needs and Disabilities
SES	Socioeconomic status
SKIMA	Subject Knowledge in Mathematics (Rowland, Martyn, Barber, & Heal, 2001)
SSP	School Support Programme
TAL	Tussendoelen Annex Leerlijnen (In Dutch); Intermediate Attainment Targets (in English)
TIMSS	Trends in International Mathematics and Science Study

Table of Contents

Executive Summary	7
A View of Mathematics	8
Context	8
Pedagogy	9
Practices	9
Curriculum Development	10
Curricular Issues	11
Partnership with Parents	12
Teacher Preparation and Development	12
Key Implications	13
Introduction	17
Pedagogy	20
Curriculum	21
Curricular Supports	22
Chapter 1: Good Mathematics Pedagogy	23
Starting with Play	24
Principles that Emphasise People, Relationships and the Learning Environment	26
Principles that Emphasise Learning	27
Engaging Children’s Preconceptions	28
Integrating Factual Knowledge and Conceptual Frameworks	29
Promoting a Metacognitive Approach	30
Features of Good Mathematics Pedagogy	31
Conclusion	33
Chapter 2: Teaching Practices	35
Meta-Practices	36
Promotion of Math Talk	37
Development of a Productive Disposition	40

<u>Emphasis on Mathematical Modeling</u>	41
<u>Cognitively Challenging Tasks</u>	43
<u>Formative Assessment</u>	44
<u>Practices in Integrative Contexts</u>	45
<u>Play</u>	46
<u>Story/Picture-Book Reading</u>	48
<u>Project Work</u>	53
<u>Learning Mathematics through the Arts and Physical Education</u>	55
<u>Digital Tools</u>	58
<u>Conclusion</u>	62
Chapter 3: Curriculum Development	63
<u>Curriculum Aims</u>	65
<u>Curriculum Goals</u>	65
<u>Mathematical Processes</u>	66
<u>Communicating</u>	66
<u>Reasoning</u>	67
<u>Argumentation</u>	68
<u>Justifying</u>	68
<u>Generalising</u>	68
<u>Representing</u>	69
<u>Problem-Solving</u>	70
<u>Connecting</u>	70
<u>Content Areas</u>	71
<u>Number</u>	72
<u>Measurement</u>	74
<u>Geometry and Spatial Thinking</u>	76
<u>Algebraic Thinking</u>	78
<u>Data and Chance</u>	80
<u>Content Areas and Curriculum Presentation</u>	80
<u>Conclusion</u>	82
Chapter 4: Curricular Issues	85
<u>An Equitable Curriculum</u>	86

Exceptional Children	87
Children with Intellectual and Developmental Difficulties	87
Children with Hearing Impairment	87
Children with Visual Impairment	88
Children with Autistic Spectrum Disorders	88
Mathematically-talented Children	88
Children in Culturally Diverse Contexts	89
English Language Learners	89
Children Learning Mathematics through Irish	90
Children in Socioeconomically Disadvantaged Contexts	91
Early Intervention	93
Allocation of Time to Teaching Mathematics	93
Preschool Settings	94
Primary School Settings	95
Emphasis on Different Mathematics Content Areas	97
Mathematics Across the Curriculum	98
Conclusion	100
Chapter 5: Partnership with Parents	101
Parents and their Children’s Mathematical Learning	102
Communicating with Parents about Mathematics	104
Sharing Information with Parents	105
A Two-way Flow of Information	105
Technology and Communicating with Parents	107
Parents and Children Discuss Mathematics	107
Parents and Teachers Collaborating about Mathematics Learning	108
Parent and Child Collaborating about Mathematics	110
Conclusion	111
Chapter 6: Teacher Preparation and Development	113
The Goal of Mathematics Teacher Preparation	115
Mathematical Knowledge for Teaching (MKT)	115
MKT Research in Ireland	116
Profound Understanding of Fundamental Mathematics	116

'Doing' Mathematics.	117
Frameworks for Thinking about Pedagogy.	118
The Knowledge Quartet.	118
Effective Teachers' Framework	121
Using Tools for Teacher Preparation.	121
Mathematics Teacher Development (CPD).	122
Conclusion	123
Chapter 7: Key Implications.	125
Glossary	129
References	133

The executive summaries of reports No. 17 and No. 18 are available online at ncca.ie/primarymaths. The online versions include some hyperlinks which appear as text on dotted lines in this print copy.

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Executive Summary



The review of research on mathematics learning of children aged 3–8 years is presented in two reports. These are part of the NCCA's Research Report Series (ISSN 1649–3362). The first report (Research Report No. 17) focuses on theoretical aspects underpinning the development of mathematics education for young children. The second report (Research Report No. 18) is concerned with related pedagogical implications. The key messages from Report No. 18 are presented in this Executive Summary.

A View of Mathematics

Both volumes are underpinned by a view of mathematics espoused by Hersh (1997): mathematics as 'a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context' (p. xi). Mathematics is viewed not only as useful and as a way of thinking, seeing and organising the world, but also as aesthetic and worthy of pursuit in its own right (Zevenbergen, Dole, & Wright, 2004). All children are viewed as having an ability to solve mathematical problems, make sense of the world using mathematics, and communicate their mathematical thinking. This shift in perspective demands a change in pedagogy – in particular it puts the teaching-learning relationship at the heart of mathematics.

Context

In Report No. 17 we argue that the overall aim of the curriculum should be the development of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) (National Research Council [NRC], 2001). As mathematization plays a central role in developing proficiency, the processes of mathematization should permeate all learning and teaching activities. These include connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising. Foregrounding mathematical proficiency as the aim of mathematics education has the potential to change the kind of mathematics and mathematical learning that young children experience. As it demands significant changes in pedagogy, curriculum and curricular supports (Anthony & Walshaw, 2007), it also poses challenges that are wide-ranging and systemic.

The development of mathematical proficiency begins in the preschool years, and individuals become increasingly mathematically proficient over their years in educational settings. This implies that educators in the range of early childhood settings need to develop effective pedagogical practices that engage learners in high-quality mathematics experiences. There is a concomitant need to address issues related to curriculum content and presentation. In particular, the questions of how to

develop a coherent curriculum and how to formulate progressions in key aspects of mathematics are important. The view of curriculum presented in this report is both wide and dynamic. It is recognised that the mathematics education of young children extends beyond the walls of the classroom: family and the wider community can make a significant contribution to children's mathematical achievement (e.g., Sheldon & Epstein, 2005).

Pedagogy

It is impossible to think about good mathematics pedagogy for children aged 3–8 years without acknowledging that much early mathematical learning occurs in the context of children's play (e.g. Seo & Ginsburg, 2004). Educators need to understand how mathematics learning is promoted by young children's engagement in play, and how best they can support that learning. For instance, adults can help children to maximise their learning by helping them to represent and reflect on their experiences (e.g., Perry & Dockett, 2007a). Learning through play is seen as fundamental to good mathematics pedagogy in early childhood. It assumes varying degrees of emphasis depending on the age of the child. Recent research points to a number of other important principles which underpin good mathematics pedagogy for children aged 3–8 years (e.g., Anthony & Walshaw, 2009a; NRC, 2005). These principles focus on people and relationships, the learning environment and learners. Features of good mathematics pedagogy can be identified with reference to these principles. Both the principles and the features of pedagogy are consistent with the aim of helping children to develop mathematical proficiency. They pertain to all early educational settings, and are important in promoting continuity in pedagogical approaches across settings.

Practices

Good mathematics pedagogy incorporates a number of meta-practices (i.e., overarching practices) including the promotion of math talk, the development of a productive disposition, an emphasis on mathematical modeling, the use of cognitively challenging tasks, and formative assessment. The literature offers a range of perspectives, and advice, as to the issues for educators in integrating these elements into their practices. In doing so, the vision of 'mathematics for all' is supported.

Good mathematics pedagogy can be enacted when educators engage children in a variety of mathematically-related activities across different areas of learning. The activities should arise from children's interests, questions, concerns and everyday experiences. A deep understanding of the features of good pedagogy should inform the ways in which educators engage children in mathematically-related activities such as play, story/picture-book reading, project work, the arts and physical education. The potential of these activities for developing mathematical proficiency can best be realised when educators focus on children's mathematical sense-making. In addition, educators need to maximise the opportunities afforded by a range of tools, including digital tools, to mediate learning.

Curriculum Development

Goals, coherent with the aim of mathematical proficiency, should be identified. These goals relate both to process and content. The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded. In line with the principle of ‘mathematics for all’, each of the five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance – should be given appropriate attention.

Goals need to be broken down for planning, teaching and assessment purposes. Learning paths can be helpful for this purpose. As is outlined in Report No. 17, differences in the ways learning paths are presented in the literature rest largely on their theoretical underpinnings. For example, developmental progressions described by Sarama and Clements (2009) are finely grained and age-related, whereas the TAL¹ trajectories developed in the context of Realistic Mathematics Education (van den Heuvel-Panhuizen, 2008) are characterised by fluidity and the role of context. In line with a sociocultural approach to the learning of mathematics, we advocate that learning paths be used in a flexible way to posit shifts in mathematical reasoning, i.e. critical ideas in each of the domains. Narrative descriptors of critical ideas can be used to inform planning and assessment. Learning outcomes, relating to content domains and processes, can then be derived from a consideration of the goals, learning paths and narrative descriptors. The figure below shows an emerging curriculum model highlighting how the relationships between the different elements may be conceptualised.

1 In Dutch, learning-teaching trajectories are referred to as TALs (i.e., Tussendoelen Annex Leerlijnen).

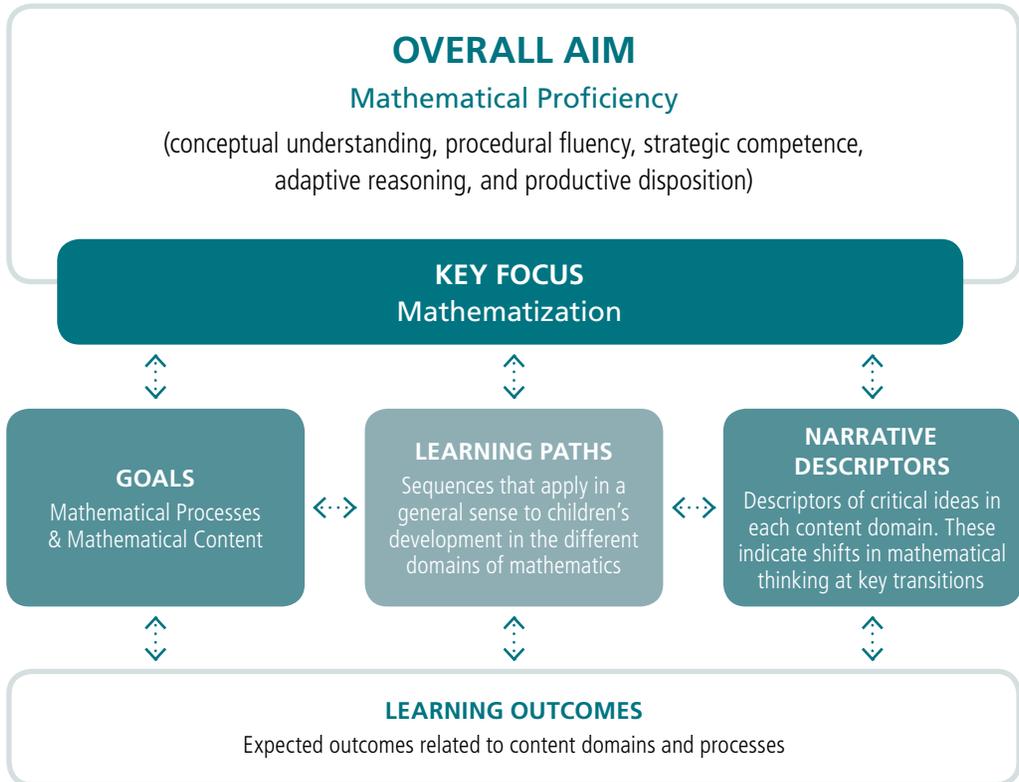


Figure ES.1: Emerging Curriculum Model

Curricular Issues

While the specification of processes and content in the mathematics curriculum is critically important, attention should also be given to issues that relate to curriculum access and curriculum implementation. This is based on the premise that the curriculum must serve all children, including exceptional children (those with developmental delays and those with exceptional talent) and children in culturally diverse contexts. Other key issues include the timing of early intervention, the allocation of time to mathematics in early learning settings, and how best to achieve the integration of mathematics across the curriculum.

Consistent with Lewis and Norwich's (2005) concept of continua of common teaching approaches that can be subject to varying degrees of intensity depending on children's needs, modifications to the mathematics curriculum for children with special education needs are proposed. Mathematically-talented children should be supported in deepening their understanding of and engagement with the existing

curriculum rather than being provided with an alternative one. In the case of English-language learners, and children attending Irish-medium schools, the key role of mathematical discourse and associated strategies in enabling access to the language in which the curriculum is taught are emphasised (e.g., Chapin, O'Connor, & Anderson, 2009). Attention to language is also highlighted as a critical issue in raising the mathematics achievement of children in DEIS schools. More generally, it is noted that there is now strong research indicating that additional support should be provided at an earlier stage than is indicated in current policy documents (e.g. Dowker, 2004; 2009). There is a need to allocate sustained time to mathematics to ensure that all children engage in mathematization. Dedicated and integrated time provision is recommended. The value of integrating mathematics across areas of learning is recognised, though it is acknowledged that relatively little research is available on how best to achieve this.

Partnership with Parents

In line with the emphasis on parental involvement in the *National Strategy to Improve Literacy and Numeracy among Children and Young People (2011–2020)* (Department of Education and Skills [DES], 2011a), the key role of parents in supporting children to engage in mathematics is emphasised. There is a range of activities in which parents can engage with schools so that both parents and educators better understand children's mathematics learning. However, it is acknowledged that research on parental involvement in mathematics lags behind similar research relating to parental involvement in reading literacy.

In the literature on parental involvement, the need to establish a continuous, two-way flow of information about children's mathematics learning between educators and parents is a key theme. There is potential for technology to support this. Strategies designed to support parents to better understand their child's mathematical learning include observation of and discussion on children's engagement in mathematical activities in education settings. Mechanisms are required to inform parents about the importance of mathematics learning in the early years, and what constitutes mathematical activity and learning for young children. The significant role that parents play in the mathematical development of their children should be foregrounded.

Teacher Preparation and Development

Curriculum redevelopment is strongly contingent on parallel developments in pre-service and in-service education for educators across the range of settings. In particular, professional development programmes need to focus on the features of good mathematics pedagogy and the important meta-practices that arise from these.

In order for teachers to foster mathematical proficiency in children, they themselves need to be mathematically proficient. Therefore, teacher preparation courses need to provide opportunities

for pre-service teachers to engage in rich mathematical tasks. Educators need to develop mathematical knowledge for teaching through a collaborative focus on teaching and learning of mathematics. They need opportunities to notice children's engagement in mathematics and responses to mathematical ideas. Case studies of practice are valuable tools in this regard. These can be used by pre-service (and in-service) teachers to question and critique the practice of others in order to develop 'local knowledge of practice' (Cochran-Smith, 2012, p. 46).

Among the recommendations for the continuing professional development of teachers (CPD) is investment in stronger systems of clinical supervision across the preparation-induction boundary (Grossman, 2010). The notion of clinical supervision could mean an emphasis on developing good mathematics teaching practices through collaborative review and reflection on existing practice. This is important because inquiry as a stance has been advocated as a successful key to teacher change (Jaworski, 2006). In this regard, lesson study is a practice that is currently foregrounded in the literature as a significant development in school-based professional development (e.g., Corcoran & Pepperell, 2011; Fernández, 2005). In lesson study, publicly available records of practice or 'actionable artifacts' are important by-products (Lewis, Perry, & Murata, 2006, p. 6). The practice offers opportunities at school and classroom level for enactment of critical inquiry into mathematics lessons.

Key Implications

The key implications for the redevelopment of the mathematics curriculum arising from the review of research presented in this report are as follows:

- The curriculum should be coherent in terms of aims, goals relating to both processes and content, and pedagogy. ([Chapter 1](#), [Chapter 3](#))
- The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded in curriculum documentation and should be central to the mathematical experiences of all children. ([Chapter 2](#), [Chapter 3](#))
- The redeveloped mathematics curriculum needs to acknowledge and build on the pedagogical emphases in *Aistear*. ([Chapter 2](#))
- In order to facilitate transitions, educators across early education settings need to communicate about children's mathematical experiences and the features of pedagogy that support children's learning. ([Chapter 1](#))
- The principles and features of good mathematics pedagogy as they pertain to people and relationships, the learning environment, and the learner, should be emphasised. ([Chapter 1](#))
- The overarching meta-practices and the ways in which they permeate learning activities should be clearly explicated. ([Chapter 2](#))

- Educators should be supported in the design and development of rich and challenging mathematical tasks that are appropriate to their children’s learning needs. ([Chapter 2](#), [Chapter 5](#))
- The curriculum should exemplify how tools, including digital tools, can enhance mathematics learning. ([Chapter 2](#))
- Children should engage with all five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance. The strand of Early Mathematical Activities as presented in the current PSMC should be integrated into the five content areas. ([Chapter 3](#))
- In curriculum documentation, critical ideas in each content domain need to be explicated and expressed as narrative descriptors. These critical ideas, derived from learning paths, should serve as reference points for planning and assessment. In presenting these ideas, over-specification should be avoided. Learning outcomes arising from these also need to be articulated. ([Chapter 3](#))
- Narrative descriptors of mathematical development, that is, descriptions of critical ideas, should be developed in class bands, e.g., two years. These critical ideas indicate shifts in children’s mathematical reasoning in each of the content domains. ([Chapter 3](#))
- The principles of equity and access should underpin the redeveloped mathematics curriculum. The nature of support that enables exceptional children (those with developmental delays and those with exceptional talent), children in culturally diverse contexts and children in disadvantaged circumstances to experience rich and engaging mathematics should be specified. ([Chapter 4](#))
- Additional support/intervention for children at risk of mathematical difficulties should begin at a much earlier point than is specified in the current guidelines. ([Chapter 4](#))
- Learning outcomes in mathematics should be cross-referenced with other areas of learning and vice-versa, in order to facilitate integration across the curriculum. ([Chapter 2](#), [Chapter 4](#))
- Additional time allocated for mathematics should reflect the increased emphases on mathematization and its associated processes. Some of this additional time might result from integration of mathematics across areas of learning. While integration has the potential to develop deep mathematical understanding, the challenges that it poses to teachers must be recognised. ([Chapter 3](#), [Chapter 4](#))
- Ongoing communication and dialogue with parents and the wider community should focus on the importance of mathematics learning in the early years, the goals of the mathematics curriculum and ways in which children can be supported to achieve these goals. ([Chapter 5](#))
- Structures should be put in place that encourage and enable the development of mathematical knowledge for pre-service and in-service teachers. Educators need to be informed about goals, learning paths and critical ideas. Records of practice, to be used as a basis for inquiry into children’s mathematical learning and thinking, need to be developed. ([Chapter 6](#))

- Educators need to be given opportunities to interrogate and negotiate the redeveloped curriculum with colleagues as it relates to their setting and context. Time needs to be made available to educators to engage in collaborative practices such as lesson study. ([Chapter 6](#))
- Given the complexities involved, it is imperative that all educators of children aged 3–8 years develop the knowledge, skills, and dispositions required to teach mathematics well. ([Chapter 6](#))
- Given the central importance of mathematics learning in early childhood and as a foundation for later development, mathematics should be accorded a high priority, at both policy and school levels, similar to that accorded to literacy. ([Chapter 4](#), [Chapter 5](#))



Introduction



In Report No. 17, we identified a number of implications for mathematics pedagogy and curriculum for children aged 3–8 years that arise from the research literature. These include the following:

- The curriculum should present a view of all children as having the capacity to engage with deep and challenging mathematical ideas and processes from birth.
- The overall aim of the curriculum should be the development of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition). As mathematization plays a central role in developing proficiency, the processes of mathematization should permeate all learning and teaching activities. These include connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising.
- The curriculum should foreground mathematics learning and development as dependent on children's active participation in social and cultural experiences, while also recognising the role of internal processes.
- In line with the theoretical framework underpinning the proposed curriculum, mathematical discourse (math talk) should be integral to the learning and teaching process.
- The goal statements of the curriculum should be aligned with its underlying theory. An approach whereby processes are foregrounded, but content areas are also specified, is consistent with a participatory approach to mathematics learning and development. In the curriculum, general goals need to be broken down for planning, teaching and assessment purposes. Critical ideas indicating the shifts in mathematical reasoning required for the development of key concepts should be identified.
- Based on the research which indicates that teachers' understanding of developmental progressions (learning paths) can help them with planning, educators should have access to information on general learning paths for the different domains.
- The curriculum should foreground formative assessment as the main approach for assessing young children's mathematical learning, with particular emphasis on children's exercise of agency and their growing identities as mathematicians.
- A key tenet of the curriculum should be 'mathematics for all'. Central to this is the vision of a multicultural curriculum which values the many ways in which children make sense of mathematics.

This report explores these implications. As in Report No. 17, it is premised on a view of mathematics as not only useful and a way of thinking, seeing and organising the world, but also as aesthetic and worthy of pursuit in its own right (Zevenbergen et al., 2004). It recognises that mathematics is ‘power’, often acting as a ‘gatekeeper to social success’ (Gates & Vistro-Yu, 2003, p. 32), in that individuals who do well in the subject are likely to have better access to jobs, college courses and higher incomes than those who do not (Dooley & Corcoran, 2007). It is also premised on the view that mathematics is not absolute and certain but is constructed by a community of learners. In the words of Hersh (1997, p. xi), mathematics is ‘a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context’. This report is thus also founded on a view of all individuals having an innate ability to solve problems and make sense of the world. This shift in perspective demands a change in pedagogy – in particular it puts the teaching-learning relationship at the heart of mathematics.

While the view of mathematics as a problem-solving activity permeates the 1999 Primary School Mathematics Curriculum (PSMC) (Government of Ireland, 1999a), there is considerable evidence of a mismatch between this view and the mathematics that the majority of children experience in the classroom. In a survey conducted by the Inspectorate of the then Department of Education and Science (DES) (DES, 2005a) of 61 classes, it was found that there was an over-reliance on traditional textbook problems in almost one third of classes. National surveys of mathematics confirm this situation – indeed textbooks are used by most primary school children on a daily basis and they act as the main planning tool for the majority of teachers in second, fourth and sixth classes (Eivers, et al., 2010; Shiel, Surgenor, Close, & Millar, 2006). Moreover, findings of studies conducted by Dunphy (2009) and Murphy (2004) suggest a strict adherence to textbook activities by many teachers of junior infant and senior infant classes respectively. The continued emphasis on lower-order thinking skills and mathematical procedures in Irish schools means that many children are being denied the opportunity to experience mathematics as a creative and engaging process. Ford and Harris (1992) suggest that the creativity that is innate to young preschool children is inhibited by a school system that rewards convergent thinking and approves correct answers. Barnes (2000) contends that students need to experience excitement and work with uncertainty in order for moments of insight to occur and suggests that

[I]f instruction progresses by small, simple steps, and the teacher anticipates difficulties and provides immediate clarification, students will have less need to struggle and less occasion to make efforts of their own to achieve understanding and insight. (p. 41)

Sinclair and Watson (2001) make a similar argument:

Learners may need to be inducted into the wonder of mathematics, to experience wonder vicariously through the teacher (including the stages of pleasure and frustration that sense-making requires) and, more urgently, to set aside the illusion of mathematics as systematic knowledge so complete that there is nothing more to expect. (p. 41)

While the worked examples that tend to predominate textbook pages may serve a purpose in promoting procedural fluency, they are generally not helpful in providing children with opportunities to develop the other strands of mathematical proficiency and to experience the excitement and wonder of mathematics.

Foregrounding mathematical proficiency as the aim of mathematics education has the potential to change the kind of mathematics and mathematical learning that young children experience. It also poses challenges that are wide-ranging and systemic. Anthony and Walshaw (2007), in their 'best evidence synthesis' about what pedagogical approaches work to improve children's learning of mathematics, say of the mathematical proficiency strands:

These are the characteristics of an apprentice user and maker of mathematics and are appropriated by the student through effective classroom processes. They incorporate curriculum content, classroom organisational structures, instructional and assessment strategies, and classroom discourse regarding what mathematics is, how and why it is to be learned and who can learn it. (p. 5)

It follows that changes in the mathematics experienced by young children demand significant changes in pedagogy, curriculum and curricular supports, each of which is addressed in this volume.

Pedagogy

A decade or so ago it was observed that we just didn't know enough about the issue of effectively supporting early mathematics learning across the age range 3–8 years (e.g. Gifford 2004; Ginsburg & Golbeck, 2004). It is still the case that mathematics teaching and learning in the prior-to-school years is under-researched (e.g., Anthony & Walshaw, 2009a). In Report No. 17 we identified mathematical proficiency as a key aim of mathematics education. The strands are interwoven and interdependent and each of the strands becomes progressively more developed in children as their mathematical experiences become increasingly sophisticated. As children develop proficiency in one strand, there are developments in other strands also. As we understand it, the development of mathematical proficiency begins in the preschool years, and individuals become increasingly mathematically proficient over their years in educational settings. As pointed out above, the strands develop as a result of good pedagogy. In this volume, the key issues related to good pedagogy are examined.

In [Chapter 1](#) a number of important principles which underpin the features of good mathematics pedagogy for young children are identified. These principles focus on people and relationships, the learning environment, and the learner. Arising from these principles, we present features of good mathematics pedagogy.

We survey the literature on key overarching pedagogical practices – math talk, disposition, modeling, tasks and assessment – in [Chapter 2](#). We also give attention to a variety of mathematically-related activities across different areas of learning. We argue that these activities should arise from children’s interests, questions, concerns and everyday experiences. Furthermore, educators need to focus on children’s mathematical sense-making so that the strands of mathematical proficiency are developed.

Curriculum

It is now widely acknowledged that along with addressing pedagogy, there is a concomitant need to address issues related to curriculum content and presentation. In terms of curriculum redevelopment in Ireland, the question of how to formulate progressions in key aspects of mathematics arises. Some observations can be made about the presentation of content in the 1999 PSMC (Government of Ireland, 1999a). Here we find a large number of content objectives for each class level, e.g., 55 objectives for first class and 60 for second class (Murata, 2011). This specificity is reflected across all years, including the most junior classes. Murata argues that while this may be helpful in supporting educators to make connections across classes, it also carries the risk that educators see learning of mathematics as made up of mastery of discrete units without connections. Furthermore, the listing of the objectives as a list of competencies (e.g., all content objectives are preceded by the phrase, ‘The child should be enabled to ...’) results in a reduction of content to basics and in frequent testing to decide on individual children’s levels of mastery. In this volume, we consider ways in which the mathematics curriculum for 3–8 year olds might be redeveloped and represented.

In [Chapter 3](#), we address the development of a coherent curriculum, where there is close alignment between the aim of mathematical proficiency and goals related to processes and content. We describe each of the processes associated with mathematization. We identify key issues in content areas related to Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance. We also examine how curricula in other jurisdictions are presented.

We draw attention to some important curricular issues in [Chapter 4](#). In particular, we explore the idea of an equitable curriculum to which all children, including exceptional children and children in culturally diverse contexts, have access. In order that all children have access to powerful mathematical ideas, there are other questions that need to be addressed. Among them are: the timing of early intervention in mathematics, the allocation of time to mathematics in early learning settings, and the integration of mathematics across the curriculum. We present some findings on how such questions might best be addressed.

Curricular Supports

Remillard (1999) suggests that there are two levels to curriculum development – one is the conceptualisation of plans and the development of resources for teachers; the second is what teachers ‘do’ to implement these plans in their classrooms. A teacher, therefore, is integral to curriculum development. It is also recognised that the mathematics education of young children extends beyond the walls of the classroom – family and the wider community can make a significant contribution to children’s mathematical achievement (e.g., Sheldon & Epstein, 2005). Therefore attention is also given in this report to the ways that teachers and parents can be supported to cultivate a rich mathematical learning environment for 3- to 8-year-old learners.

In [Chapter 5](#) we look at the importance of parents engaging in discussion with their child about mathematically-related activities that arise in the home, and in the context of homework when appropriate. Collaboration and sharing of information between parents and teachers are highlighted. Reference is made to various initiatives involving parents and local communities.

In [Chapter 6](#), we explore issues related to teacher preparation and development. If pre-service and in-service educators of young children are to promote good mathematics learning, they must have a strong working knowledge of mathematics, and an openness to and facility with the processes of mathematization. We examine the construct of Mathematical Knowledge for Teaching (MKT) which is integral to teacher preparation and development. The need to develop MKT through critical and collaborative inquiry is addressed and different models of professional development are described.

In [Chapter 7](#), we outline key implications for the redevelopment of the PSMC for children for 3–8 years of age arising from this volume.

CHAPTER 1

Good Mathematics Pedagogy



Pedagogy has been defined as ‘...the deliberate process of cultivating development’ (Bowman, Donovan, & Burns, 2001, p. 182). A high degree of direct adult engagement and strong guidance is implicated in this definition, and such engagement is particularly necessary in relation to mathematics learning and teaching (e.g., Ginsburg et al., 2005). In pedagogical terms, the educator engages in practices that promote and assess early mathematics learning. Siraj-Blatchford, Sylva, Muttock, Gilden & Bell, (2002, p. 27) describe pedagogy as ‘the practice (or the art, the science or the craft) of teaching’ but they also point out that any adequate conception of educational practices for young children must be wide enough to include the provision of learning environments for play and exploration.

Starting with Play

Children’s play and interests are the foundation of their first mathematical experiences. Play and playful activities/situations provide the main contexts in which most of children’s prior- to-school mathematics learning takes place (e.g., Seo & Ginsburg, 2004; Van Oers, 2010). We know that children in their free play spontaneously engage in a great deal of mathematics, some of it at levels that are quite advanced. Sometimes they may play with mathematics itself (e.g., Ginsburg, Inoue, & Seo, 1999). Child-initiated play is central to the activity of young children and much mathematical learning occurs within the play environment (Montague-Smith & Price, 2012; Moyles, 2005). These play experiences become mathematical as children represent and reflect on them (Sarama & Clements, 2009).

Play provides a context wherein children can reflect on their past experiences, make connections across experiences, represent these experiences in different ways, explore possibilities and create meaning. These processes of play have strong connections to mathematical thinking (Perry & Dockett, 2007a). Play is a rich context for the promotion of mathematical language and concepts.

The adults around the child are often unaware of the child’s engagement with mathematical ideas, and may not generally recognise this engagement and how it arises from children’s spontaneous interests in, and exploration of, the world around them (Ginsburg, 2009a). Play is a context within which children can explore their mathematical ideas but it also provides a context within which adults can support and develop children’s ideas. The adults with whom children interact have a critical role in helping children to reflect on (and talk about) their experiences in play and so to maximise the learning potential. Sensitive structuring of children’s play can be effective in promoting

mathematical thinking and learning (e.g., Ginsburg, et. al, 1999; Perry & Dockett, 2007a; Pound, 2008). From this perspective, learning through play is seen as fundamental to good mathematics pedagogy in early childhood. It assumes varying degrees of emphasis depending on the age of the child.

The potential of play to provide a 'bridging tool to school' is very significant (Broström, 2005). In recognising this potential, teachers need to integrate mathematics learning within children's play activity. For instance, the incremental development of children's spatial-geometric reasoning and their geometric and measurement skills across the transition period can be achieved through a systematic approach to the teaching of related concepts. This approach allows for the integration of problem-solving skills and content knowledge (Casey, 2009). Play with blocks provides the context within which teachers can teach the key aspects of spatial reasoning. Children's early experiences with blocks includes open-ended play, but over a period of time teacher-guided activities can serve to focus the children on sequenced spatial problems. As children's experiences with the blocks grow, and as they engage in various problem-solving activities, initial concepts are strengthened. These are then extended in later activities. As children solve mathematically-related problems they should be encouraged to use a range of informal approaches and problem-solving strategies with the intention of guiding them, as their understanding increases, towards the most effective strategies; they should be encouraged to talk about and compare their strategies with those used by others and learning experiences should target critical ideas (i.e. where conceptual shifts are required) (Fuson, Kalchman, & Bransford, 2005).

As stated in Report No. 17 (see Chapter 1, Section: [*Defining Mathematics Education for Children Aged 3–8 years*](#)), there are now some key sources that educators can look to for guidance in relation to teaching practices. These include statements from *The National Association of Educators of Young Children* (NAEYC) in the United States, who joined forces with the *National Council of Teachers of Mathematics* (NCTM) to issue a position paper (NAEYC/NCTM, 2002/2010) on early childhood mathematics. Similarly, in Australia, *Early Childhood Australia* and the *Australian Association of Mathematics Teachers* set out their position on pedagogical practices for early childhood mathematics (AAMT/ECA, 2006). These two sets of recommendations share a concern to engage children in appropriate and sensitive ways in what are to them interesting, meaningful, challenging and worthwhile mathematical experiences. They promote a pedagogy which is interactive, engaging and supportive of all children's learning. They also provide an overarching framework for considering what is important in early mathematics pedagogy across early education settings.

In this chapter we explore the features of good mathematics pedagogy. But in order to derive these we first examine the principles underpinning good mathematics pedagogy. We consider two sets of principles, both of which explicitly place mathematical proficiency at the core of mathematics education. The first set of principles arises from the work of Anthony and Walshaw (2007; 2009a) from New Zealand, who synthesised available international research on effective pedagogy in mathematics. These principles emphasise people, relationships, and the learning environment. The second set of principles is from the United States and relates to the learner. It arises from the NRC Report (2005)

which focused on how principles of learning can be applied to mathematics education. These sets of principles present two viewpoints on how mathematical proficiency can be supported by educators, and, in the discussion which follows, we bring them together to offer a comprehensive account of the features of good mathematics pedagogy.

Principles that Emphasise People, Relationships and the Learning Environment

Anthony and Walshaw (2007; 2009a) draw together available evidence about what pedagogical practices work to improve children's outcomes in mathematics. In doing so, they stress the importance of interrelationships, and the development of community in the classroom.² Arising from their research, they present the following principles that characterise effective settings and effective educators:

- an acknowledgement that all children, irrespective of age, have the capacity to become powerful mathematics learners
- a commitment to maximise access to mathematics
- empowerment of all to develop positive mathematical identities and knowledge
- holistic development for productive citizenship through mathematics
- relationships and the connectedness of both people and ideas
- interpersonal respect and sensitivity
- fairness and consistency (Anthony & Walshaw, 2007, p. 1).

Cobb (2007) observes that the image of effective pedagogy that emerges from Anthony and Walshaw's synthesis is that of teaching as a coherent system rather than a set of discrete, interchangeable strategies. This pedagogical system encompasses four elements that work together as a set of connected parts:

- a non-threatening classroom atmosphere
- instructional tasks
- tools and representations
- classroom discourse.

2 In this report we use the term classroom similar to the way in which it is used in the NAEYC/NCTM position statement (2002/2012), i.e. as referring to any group setting for 3- to 8-year-old children (e.g. preschool, family child care, primary school).

Anthony and Walshaw identify a range of features of effective pedagogy based on people, relationships and the learning environment. These features are seen to enhance the development of young children's mathematical identities, dispositions and competencies. For instance, they include a balance between teacher-directed/initiated and child-directed/initiated activity and a focus on appropriate relationships. Everyday activity, including play, is seen to provide a rich context for learning but, as they observe, 'unstructured play, by itself, is unlikely to provide sufficient support for young children's mathematical development' (p. 110). Their findings indicate that providing for children's optimum development through their access to explicitly mathematical experiences, and for their engagement in interactions which support and extend their mathematics learning, are both critical dimensions of the learning environment. The development of an increased focus on mathematical activities, games, books and technology are some of the experiences that are seen to enhance opportunities for learning. The research indicates that educators' increased mathematical awareness enables them to recognise and respond to opportunities for developing all children's ideas and for enhancing mathematics learning. The importance of responsive pedagogical interactions by adults with children is foregrounded, as is the engagement of children in discussions which promote their abilities to express their thinking and to conjecture, predict and verify. Differences in home experiences of children, with some families more orientated to mathematics than others, is seen as an issue of which educators need to be aware. The necessity of acknowledging children's mathematics learning in the home and the community and of working with families to understand and build on this, is emphasised as a key aspect of pedagogy. Anthony and Walshaw argue that mathematical proficiency is appropriated by children as they engage in the range of learning interactions described above. The main features of good mathematics pedagogy arising from both Anthony and Walshaw's report and that of the NRC, discussed below, are displayed in Table 1.1.

Principles that Emphasise Learning

Three principles which were derived from a synthesis of work on learning are considered from the perspective of mathematics education (NRC, 2005). The principles are as follows:

- teachers must engage children's preconceptions
- understanding of mathematics requires factual knowledge and conceptual frameworks
- a metacognitive approach enables children to monitor their own learning and development.

While these principles focus on individual learners, we saw above that those presented by Anthony and Walshaw focus more on the context of learning. As such, the two sets can be seen as complementary. Fuson et al. (2005) argue that the strands of mathematical proficiency map directly to the NRC principles:

Principle 2 argues for a foundation of factual knowledge (procedural fluency) tied to a conceptual framework (conceptual understanding), and organised in a way to facilitate retrieval and problem solving (strategic competence). Metacognition and adaptive reasoning both describe the phenomenon of ongoing sense making, reflection, and explanation to oneself and others...the preconceptions students bring to the study of mathematics affect more than their understanding and problem solving; those preconceptions also play a major role in whether students have a productive disposition towards mathematics, as do, of course, their experiences in learning mathematics. (p. 218)

As teachers work with these principles, children's mathematical proficiency is supported. Below we discuss the pedagogical features implied by each of these three principles.

Engaging Children's Preconceptions

The first NRC principle relates to the belief that all new understandings are constructed on a foundation of existing understandings and experiences. One implication of this is that educators must familiarise themselves with children's early mathematics experiences and understandings. We know that there is great diversity in these experiences and understandings (e.g., Tudge & Doucet, 2004), and this can present considerable challenges for teachers, especially at key transitional points such as entry to preschool or school. This suggests a pedagogy which enables educators to find out about children's experiences (individual and collective, as appropriate) and to consider how these influence subsequent learning. A second implication is that teachers also need to ascertain, on a regular basis, children's current levels of understanding as well as their individual ways of thinking, in order to plan appropriately. Encouraging math talk (talking about mathematical thinking) is important in this regard (Fuson et al., 2005). Another implication, and in some circumstances the only option, is close observations of children's engagement in a range of learning activities, and reflection on these from a mathematical point of view (Ginsburg, 2009b). Björklund and Pramling Samuelsson (2012) identify challenges in working with the youngest children (aged 3), as those associated with taking account of children's perspectives on particular concepts and then directing their attention to more developed ways of understanding the same concept. She argues that good mathematics pedagogy is such that materials and learning objectives are specified in advance, that children have opportunities to play and explore and they are challenged with appropriate questions. As they become engaged with the materials and the activity, the educator uses the opportunity to direct children's attention and interest towards learning objectives while at the same time displaying sensitivity and responsiveness to individual children's ways of engaging with the situation.

The idea of intentionally guiding children towards effective strategies is one that needs to be emphasised for educators since it is a relatively new view of the role of educators working with the youngest children. As described in the NRC report (2009, p. 226), 'intentional teaching' is the skill of 'adapting teaching to the content, type of learning experience, and individual child with a clear learning target as a goal'. These emphases were not deemed important for 3- and 4-year-old

children until relatively recently. It is likely that emphases which take the lead from the child may present a challenge to some primary teachers also, since many of these may be more familiar with a direct instruction approach.

Integrating Factual Knowledge and Conceptual Frameworks

The second NRC principle relates to the essential role of the elements of factual knowledge and conceptual frameworks in understanding. Both factual knowledge and conceptual frameworks (organising concepts) are important and are inextricably linked. Together they promote understanding, and this, in turn, affects the ability to apply what is learned. For example, a child, from learning the count sequence, may know that four comes after three in the sequence of numbers (at this stage 'four' is still a relatively abstract and shallow concept), but through the repeated use of 'four' in diverse mathematical practices, the concept deepens and connects with other related ideas. In this instance, children's learning can be promoted when the educator intentionally guides children towards considering the range of ways in which we use the number word 'four', i.e. not just in counting but in quantifying, labelling and ordering. The educator will exploit opportunities that present themselves but will also structure activities so that curriculum goals can be promoted. Helping children to link new learning to something they already know enables them to make connections. The process of making connections is very important for young children (e.g., Perry & Dockett, 2008) and contributes towards the development of mathematical proficiency (NRC, 2001) (see Chapter 3, Section: [Connecting](#)). Counting the sides of a square helps children to connect number to geometry (NAEYC/NCTM, 2002/2010). Thus, good pedagogy emphasises both factual and conceptual knowledge.

Good pedagogy is one that engages and challenges children. It draws on learning paths to help children make progress towards curricular goals (see Report No. 17, Chapter 5, Section: [The Development of Children's Mathematical Thinking](#)). Knowing about learning paths, and the critical ideas along these, helps educators to support children's progress along the paths. It involves constructing 'bridging' activities and developing conceptual supports to help children make links between math words, written notations, quantities, operations and so on. Teachers need to understand possible preconceptions that children may hold, and they also need to be aware of possible points of difficulty. The importance of the teacher addressing these issues in a proactive way is strongly emphasised in the literature (see Report No. 17, Chapter 5, Section: [Curriculum Development and the Role of Learning Trajectories](#)). Good pedagogy emphasises the necessity of guiding children (the group and individuals) through the learning paths, ensuring a balance between learner-centred and knowledge-centred needs (Fuson et al., 2005). Ginsburg (2009b) describes how, over a number of years, children supported in this way advance beyond their informal, intuitive mathematics to develop the formal concepts, procedures and symbolism of conventional mathematics. Examples of these paths and their use in assessment are discussed in greater detail later in this report (see Chapter 2, Section: [Formative Assessment](#)) and also in Report No. 17 (see Chapter 5, Section: [Curriculum Development and the Role of Learning Trajectories](#); Chapter 6, Section: [Supporting Children's Progression with Formative Assessment](#)).

Promoting a Metacognitive Approach

The third NRC principle relates to the importance of self-monitoring, or self-regulation. Self-regulation is supported by children's ability to internally monitor and strategically control actions, as they attempt to undertake a task or solve a problem. According to Vygotsky (1978), self-regulation is promoted through interactions with more experienced others who model and articulate their successful strategies. For example, in learning to complete a jigsaw a child may first witness others (adults or peers) use strategies such as turning pieces, trying pieces, focusing on the shape, size or colour of pieces. Speech accompanying these actions may then be internalised by the child to provide self-monitoring or self-regulating strategies that can later be called on to solve similar problems. An important self-regulatory ability is that of gradually becoming able to talk oneself through similar tasks using external speech but soon moving to internal speech or abstraction (e.g., Berk & Winsler, 1995). Such an approach can help young learners take control over their own learning. One of the ways that they do this is by setting goals for themselves and by checking their own progress towards those goals.

The development of metacognitive awareness i.e. the awareness and control of one's own learning and thinking, helps children to become self-regulated learners. The recognition that children aged 3–6 years can engage in metacognitive processes is relatively recent (see Coltman, 2006 for a review of relevant literature). Organising the classroom environment and the learning activities in particular ways, with an emphasis on particular styles of discourse and interactions between adults and children and between the children themselves is critical. These factors can make a significant contribution to helping children to become independent and self-regulated learners (Whitebread, 2007). Asking children to explain their thinking contributes to the development of metacognition. As children learn to self-monitor they develop and use a meta-language to describe and express their thinking, i.e., a language that includes phrases such as 'I knew they were going to fall down' or 'I counted to see how many'. In one study in the UK (Coltman, 2006), video recordings provided undisputable evidence that children used a wide range of mathematical meta-language as they engaged with planned play activities designed to encourage mathematical talk. This was often a surprise to the educators who worked with them. Examples included 'I am going to fill all this page with numbers' (metacognitive knowledge), 'There are too many hexagons' (strategic control) and 'This is fun isn't it' (motivation). The children also showed, through their talk, an awareness of themselves and others as learners. For example, they made statements about what they were or were not able to do, or demonstrated skills such as counting to other children involved in the play. Good pedagogy, in this instance planned play activities, facilitates self-regulated mathematical learning through verbal interactions which encourage and support a focus on strategic awareness and metacognitive thinking.

Encouraging self-assessment is an important aspect of supporting self-regulation by young children, since it focuses children on thinking about cognitive processes and helps them, for example, to

identify errors and monitor thinking (see Coltman, 2006 for a review). Supporting children's self-assessment can be done through using appropriate questions. For example, in the context of constructing a particular structure using blocks, the questioning might include probes such as, 'What made you decide to make your bridge using those particular blocks? Is there any other way that you might build it?'. In relation to selecting the most efficient strategy for sharing out a purse of coins so that everyone has an equal amount, probes might include questions such as, 'Are you sure that everyone has the same amount? How do you know?' (see also Report No. 17, Chapter 6, Section: [Conversations](#); Report No. 18, Chapter 3, Section: [Reasoning](#)). Recognising mistakes, self-correcting, checking, and justifying decisions are some of the behaviours that educators can encourage and develop in order to support children towards realising their capabilities in respect of self-monitoring. These behaviours are closely related to the development of adaptive reasoning, a strand of mathematical proficiency.

The examples above indicate how the learning environment can be structured to support self-regulation. But educators can also foster a metacognitive approach by supporting children's engagement with processes such as estimation and by recognising the role that number sense plays for children learning to check on the feasibility of their responses to number-based problems (e.g., Fuson et al., 2005).

Features of Good Mathematics Pedagogy

Both the NRC report (2005) and Anthony and Walshaw's research synthesis (2007) emphasise the importance of frameworks or systems for thinking about teaching, learning and the design of learning environments. Pedagogy then is seen as a complex whole, a set of connected parts. Isolated practices are not the focus; rather, it is the way in which the different elements of the system interact that is important. Both reports emphasise the learning environment, but the latter also explicitly foregrounds people and relationships, while the former explicitly focuses on the learner.

In Table 1.1 below we list the main features of good mathematics pedagogy as drawn from a combination of both approaches. We group them under the headings of *People and Relationships*, *The Learning Environment* and *The Learner*.

Table 1.1: Features of Good Mathematics Pedagogy

People and Relationships
Strong interpersonal relationships within the setting are fundamental to children's progress.
The classroom atmosphere is one in which all children are comfortable with making contributions.
The diverse cultures of children and their families are taken seriously and treated as classroom resources.
Co-construction of mathematical knowledge is developed through the respectful discussion and exchange of ideas.
The Learning Environment
The starting point for teaching is children's current knowledge and interests.
Classroom activity and discourse focus explicitly on mathematical ideas and problems.
Tasks are designed based on children's current interests, but they also serve the long-term learning goals.
Children are given opportunities to engage in justification, argumentation and generalisation. In this way, they learn to use the language of mathematics.
A wide range of children's everyday activities, play and interests are used to engage, challenge and extend their mathematical knowledge and skills.
Learning environments that are rich in the use of a wide range of tools that support all children's mathematical learning.
Children are provided with opportunities to learn in a wide range of imaginative and real-world contexts, some of which integrate and connect mathematics with other activities and other activities with mathematics.
Investigative-type activities that stem from children's interests and questions, give rise to the creation of models of the problem which can be generalised and used in other situations.
Contexts that are rich in perceptual and social experiences are used to support the development of problem-solving and creative skills.
Children experience opportunities to learn in teacher-initiated group contexts, and also from freely-chosen but potentially instructive play activities.
The potential of everyday activities such as cooking, playing with mathematical shapes and telling the time is recognised and harnessed.
Opportunities are balanced for children to learn in small groups, in the whole-class group and individually, as appropriate.
Teaching is based on appropriate sequencing. Whilst learning paths are used to provide a general overview of the learning continua of the group of children, this is tempered with the knowledge that children do not all progress along a common developmental path.
Planned and spontaneous learning opportunities are used to promote mathematics learning.

Table 1.1: Features of Good Mathematics Pedagogy (*continued*)

The Learner
Children's reasoning is at the centre of instructional decision-making and planning.
Teaching is continually adjusted according to children's learning and as a result of on-going assessment.
Scaffolding that extends children's mathematical thinking is provided while children's contributions are simultaneously valued.
Opportunities are provided for children to engage in metacognitive-like activities such as planning and reflecting. In doing so, children are supported to set their own goals and assess their own achievements.
Assessment is carried out in the context of adult-child interactions and involves some element of sustained, shared thinking.

Ginsburg and Golbeck (2004) expressed their concern that the teaching of mathematics should be responsive to the variations in settings in which early childhood education takes place. This variation is present in terms of the profile of children attending these settings: in age, in cultural background, in language, and in ability. It is also a feature of the profile of educators working in these settings. There are also structural variations between settings, for example in group sizes. We argue that good pedagogy, the features of which are outlined above, can encourage responsiveness by diverse educators working in diverse settings to children. At the same time, identifying features provides a means of promoting good pedagogy across the age range. Anthony and Walshaw (2009a) raise the issue of continuity since they noted a disconnection between available research literature concerning preschool mathematics, and the internationally recognised mathematics education journals and conference proceedings. The authors concluded that all of this suggested a need for some bridging conversations and partnerships between and across education settings, researchers and teachers. These conversations are necessary to achieve the continuity of approaches across settings seen as centrally important for children's mathematics learning, especially at critical points such as starting school (e.g., Perry & Dockett, 2008). In Ireland, children aged 6–7 years are expected to make another transition at the point where they move from senior infants to first class and issues pertaining to this later transition are also important. Conversations between educators might focus on how the features of good pedagogy are realised in everyday activities in the different early education settings, and with children of different ages.

Conclusion

In recent research, a number of important principles (Anthony & Walshaw, 2007; NRC, 2005) which underpin the features of good mathematics pedagogy for young children have been identified. One set of principles focuses on people and relationships, and the learning environment. A complementary set of principles focuses on learning and includes the engagement of children's

preconceptions, the integration of factual knowledge and conceptual frameworks, and the promotion of a metacognitive approach. All of these principles are consistent with the aim of developing mathematical proficiency. Through combining these, we identify a comprehensive account of the features of good mathematics pedagogy. The broad sets of principles are illustrated in Fig. 1.1 below:

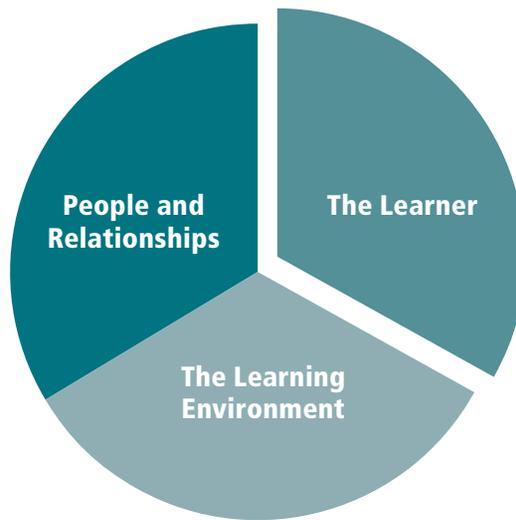


Fig. 1.1: The Connected Elements of Good Mathematics Pedagogy

However, it is impossible to think about good mathematics pedagogy for children aged 3–8 years without acknowledging that much early mathematical learning occurs within the play environment. It is also crucial to identify how adults help children maximise mathematics learning through play. Learning through play is seen as fundamental to good mathematics pedagogy in early childhood. It assumes varying degrees of emphasis depending on the age of the child. There is a gradual transition to more formal approaches as children move through primary school.

The key messages arising from this chapter are as follows:

- Educators need to understand how mathematics learning is promoted by young children’s engagement in play, and how best they can support that learning.
- The features of good mathematics pedagogy can be identified with reference to robust principles related to people and relationships, the learning environment and the learner.
- The principles and features of good mathematics pedagogy for children aged 3–8 years pertain to all early educational settings, and are important in promoting continuity in pedagogical approaches across settings.

CHAPTER 2

Teaching Practices



Guidelines for educators (e.g., NAEYC/NCTM, 2002/2010; AAMT/ECA, 2006) recommend that, for early mathematics education to be effective, teachers need to use a variety of practices and materials to support children's mathematical learning. The role adopted by the teacher is viewed as crucial. The teacher enables the learning to take place by structuring the environment and involving children in a variety of learning experiences (e.g., Pound, 1999). Successful educators build on children's interests and experiences by engaging in a wide range of teaching practices to support children's mathematical understanding. Practices highlighted later in the section (play, story/picture-book reading, project work, learning through the arts, drama and physical education, and the use and integration of tools including digital technologies) are ones that exemplify how good pedagogy is enacted in the course of everyday activities in early education settings. Each of these practices reflects a number of the features of good pedagogy as identified in Table 1.1. In addition to highlighting features of good pedagogy for educators, we have identified five overarching meta-practices that are essential in promoting mathematical thinking and understanding, and that are important in helping children towards achieving the overall aim of mathematical proficiency. These meta-practices should permeate all learning activities if optimal mathematical learning and development are to be promoted.

Meta-Practices

The five meta-practices discussed below are: promotion of math talk, development of a productive disposition, mathematical modeling, use of cognitively challenging tasks, and formative assessment. There are many others we are sure, but looking back at Report No. 17, each of these five meta-practices emerged as important in relation to pedagogy.

Promotion of Math Talk

The centrality of sustained interactions for deepening and extending children's understandings in all aspects of their learning is an issue that has received a great deal of attention in recent years, mainly due to research such as that carried out as part of the *Researching Effective Pedagogy in the Early Years* (REPEY) (Siraj-Blatchford et al., 2002). This research points to a need, in early childhood, for extended discussion with individual or small groups of children. Such opportunities create the conditions for sustained shared cognitive engagement between educator and child and for ensuring optimal cognitive challenge for all children (e.g., Anthony & Walshaw, 2009a). Skilful and thoughtful questioning of children is also a feature of pedagogy highlighted in the REPEY report, and by a number of early years' mathematics experts (e.g., Ginsburg, 1997; Copple, 2004). In Report No. 17 the issue of questioning was addressed in the context of formative assessment (see Chapter 6, Section: [Methods](#)). Skilful questioning and sensitive interventions are pedagogical strategies that have important roles to play in moving children from 'I don't know why' responses, to responses where they focus on critical aspects of the problem under consideration (e.g., Casey, 2009).

Children talking about their mathematical thinking and engaging in mathematization are identified as important ways for them to make their thinking visible (Fuson et al., 2005). It is particularly significant in supporting the growth of young children's conventional mathematical knowledge over time. Consequently its development is regarded as a key focus of early mathematics education (e.g., Ginsburg, 2009a; Perry & Dockett, 2008). However, we know that the amount of teachers' math-related talk varies, with qualitative differences in that provided by teachers in different classrooms (Klibanoff et al., 2006). As described in Report No. 17 (see Chapter 3, Section: [The Nature and Scope of Mathematical Discourse](#)), this aspect of pedagogy involves an explicit focus on language which conveys mathematical ideas related to, for example, quantity, shape, size and location. It also involves encouraging and supporting children's communication, and their initial efforts to engage in reasoning and argumentation.

The teacher has a key role to play in providing a model of the language that is appropriate in a particular mathematical context. Recasting everyday experiences using mathematical words and phrases is a key element of inducting children into talking about their mathematical thinking. Children need to be assisted in using the newly-acquired mathematical language in their descriptions, explanations and justifications. Good mathematics pedagogy recognises that some children (e.g., children living in disadvantaged circumstances; children who speak a language that is different from the language of instruction) may experience difficulty with problems presented in verbal format and there may be a need to adjust the presentation accordingly (e.g., Ginsburg, Cannon, Eisenband, & Pappas, 2006). It also takes account of the general path of development and tailors expectations of the form and extent of children's responses accordingly. Specific strategies include using children's own stories in teaching mathematics; integrating language that is familiar to children in teaching mathematics; promoting children's first language; encouraging think-aloud strategies; and integrating non-linguistic materials to facilitate maths language (Lee, Lee, & Amoro-Jimenez, 2011).

The challenge of eliciting talk about mathematics with the youngest children should not be underestimated. Many young children respond intuitively to mathematics problems but they may need support in articulating their reasoning or in justifying a solution in the conventional way (e.g., Dunphy, 2006) (see Chapter 3, Sections: [Justifying](#); [Reasoning](#)). Digital tools as ‘objects to think with’ can lead to situations wherein children externalise their thinking (see section on [Digital Tools](#) below).

Work in establishing math-talk learning communities in classroom settings (e.g., Hufferd-Ackles, Fuson, & Sherin, 2004) provides a blueprint for strengthening the focus on language as a tool for teaching and learning mathematics, and, in particular, for developing children’s understanding of concepts, strategies and mathematical representations. Such communities support children’s efforts to develop understanding, to engage in mathematical reasoning, and to communicate their mathematical ideas. Math-talk learning communities have a strong social dimension, with children sharing thoughts with others, and listening to others sharing ideas (Chapin et al., 2009). Research in the Irish primary context has documented how students build on each other’s mathematical ideas in lessons and across lessons and how a communal sense of responsibility for learning is developed (NicMhuirí, 2012).

Chapin et al. (2009) outline key teaching practices associated with improving the quality of mathematical discourse. These include:

- Using ‘talk moves’ or strategies that engage children in discourse, including revoicing (where the teacher clarifies his/her understanding of the child’s contribution), asking a child to restate someone else’s reasoning, asking a child to apply their own reasoning to someone else’s ideas, prompting for further participation, and using wait time effectively (see Dooley (2011) for examples in an Irish context).
- Using effective questioning to support key mathematical goals such as engaging children in reasoning mathematically (e.g., ‘Does this always work?’) and making connections between mathematical ideas and their application (e.g., ‘What ways have we used to solve this problem?’). It is also important to help children to rely on themselves to determine whether something is mathematically correct (e.g., ‘How do we know?’).
- Using children’s thinking to propel discussion, including identifying children’s misconceptions, enabling them to figure out those misconceptions themselves, being strategic about who shares during discussion, and choosing ideas, strategies and representations in a purposeful way that enhances the quality of the discussion.
- Setting up a supportive environment that enhances children’s engagement in mathematical discourse – e.g., by providing relevant visual aids, mathematical tools and mathematically-related vocabulary.

- Orchestrating the discourse, through such practices as anticipating children's responses to challenging mathematical tasks, monitoring their work on and engagement with tasks, selecting particular children to present their mathematical work, and connecting responses to key mathematical ideas (see also, Smith et al., 2009).

The work of Hufferd-Ackles et al. (2004) highlights the complexity of transitioning from a traditional approach to mathematics teaching, in which the teacher takes centre stage, to a discourse community, in which children make key contributions to developing their own mathematical understanding as well as that of their classmates. To support teachers in making this transition, Hufferd-Ackles et al. have produced developmental trajectories³ that address four aspects of mathematical discourse: questioning, explaining mathematical thinking, sources of mathematical ideas, and responsibility for learning. The trajectories show intermediary levels along which math-talk communities develop, and allow teachers to address difficulties and dilemmas as they move along. The levels can be summarised as follows:

- **Level 0** – teacher-directed classroom with brief answer responses from children.
- **Level 1** – teacher begins to pursue student mathematical thinking. Teacher plays central role in math-talk community.
- **Level 2** – teacher models and helps children build new roles. Some co-teaching and co-learning begins as child-to-child talk increases.
- **Level 3** – teacher functions as co-teacher and co-learner...teacher monitors all that occurs, still fully engaged. Teacher is ready to assist but now in a more peripheral and monitoring role (Hufferd-Ackles et al., 2004, pp. 88–90).

The literature points to significant challenges that teachers can encounter in implementing math-talk learning communities in their classrooms. One of these is regression to earlier levels on the trajectory when a new topic is presented as teachers may need to occupy a more central role in introducing new concepts, vocabulary or procedures (Hufferd-Ackles et al., 2004). NicMhuirí (2012), who attempted to facilitate this type of classroom community in an Irish primary classroom, also noted a tension between making student thinking an object of classroom discourse and maintaining coherent lessons and sequences of lessons (see also Fernandez, Yoshida, & Stigler, 1992). Teachers' knowledge of pedagogy for teaching mathematics is also important (see Chapter 6, Section: [A View of Mathematics](#)), since, without such knowledge, teachers may not be able to identify children's misconceptions, identify opportunities for extending their thinking and moving them along a learning path, or support them in making connections between existing knowledge and new mathematical ideas, or across aspects of mathematics (Anthony & Walshaw, 2007).

3 Unlike the trajectories/learning paths presented elsewhere in this volume, these trajectories describe development from the perspective of both teachers and children.

Dooley (2011) noted that teachers may need to reconceptualise their sense of efficacy (defined as their sense of their ability to take effective action in teaching) as they make the transition from approaches to teaching that emphasise 'telling' or 'initiative-response-evaluation' patterns, to dialogic approaches that emphasise math talk. Drawing on the work of Smith (1996), she noted that teacher efficacy could be developed, not only through generating and directing discourse (math talk), but also through selecting appropriate and relevant problems, predicting children's reasoning, and judicious or selective telling. Importantly, Dooley also noted that patterns of discourse in which teachers encourage children to explain their thinking, and focus their attention on what is not yet understood reveal greater equity in the teacher-child relationship, compared with approaches that are mainly characterised by telling or evaluation. While her work involved older primary-school children, the general principles also apply when working with younger children.

Math talk can be nurtured in a range of learning contexts including whole-group settings, small groups (e.g., collaborative learning groups) and pairs. On some occasions, teachers may assign specific tasks to groups to work on together (for example, classifying a set of shapes, solving a problem together). On other occasions, children may be asked to discuss a problem or work on the wording of an explanation in pairs, for a limited period of time. These practices allow for an increase in children's engagement in math talk, and provide the teacher with opportunities to monitor one or more groups or pairs, and gather and use information about their learning.

Development of a Productive Disposition

In Report No. 17 (see Chapter 1, Section: [*A Key Aim of Mathematics Education: Mathematical Proficiency*](#)) we saw that individuals who have a productive disposition believe that mathematics is useful and relevant and an area of learning in which they can engage successfully. Disposition has been identified as an important aspect of learning in the domain of mathematics which is acquired over time (De Corte, 2007). In early childhood, productive disposition begins with the fostering of a positive disposition towards the mathematics that they encounter in their everyday life. Bertram and Pascal (2002, p. 94) describe dispositions in early childhood as '...environmentally sensitive'. They are acquired from and affected by interactive experiences with the environment, significant adults and peers...positive dispositions are learnt but they are rarely acquired didactically'.

This implies a focus on experiences initiated by children and developed by educators (see the discussion on play and on project work below). Children play an active role in the development of their dispositions by participating and collaborating in mathematically-rich activities. Children's eagerness to participate in everyday activities such as cooking (Vandermaas-Peeler, Boomgarden, Finn, & Pittard, 2012), or shopping is an effective way of fostering positive disposition, especially in circumstances where the adult is sensitive to children's interests and preferences. As a result of her study of the number sense of 4-year-old children, Dunphy (2006) concluded that the ways in which children are engaged with mathematics, how they view mathematics and the contexts in which

mathematics are presented to them are what shape their dispositions towards mathematics. In the same study, children with a positive disposition also demonstrated a strong number sense.

A productive disposition can be fostered by educators who draw children's attention to the various aspects of mathematics, and who engage children in what to them are interesting and relevant experiences that show the usefulness of mathematics for solving everyday problems. For instance, young children starting school may already have developed a liking or enthusiasm for number, based on experiences during the preschool period, i.e., their disposition towards number is already developing (e.g., Dunphy, 2006). It is essential therefore, that children's experiences with mathematics in early education settings are ones that are engaging and challenging (e.g., see the discussion on story/picture-book reading and the mathematically-related discussion arising from this, later in the chapter). This message has important implications for the pedagogical practices used by educators. The practices used should enable children's agency and incorporate their interests and preferences.

It is important to stress that in these early years disposition is still quite malleable, and the early experiences at preschool and school are likely to be critical for some children. Hence, curriculum guidelines should emphasise that issues related to disposition (e.g., the learning environment, opportunity to participate) need to be investigated by teachers, and systematically supported so that all children can develop a productive mathematical disposition.

Emphasis on Mathematical Modeling

The idea of a mathematical model as it is generally used in mathematics education stems from the way it is used in the discipline of mathematics, that is, as a quantitative or spatial system that can be used in particular, prescribed ways. From this perspective, the model is seen as existing independently of individual or collective activity. Base-ten blocks (i.e., Dienes' blocks) that are used in the teaching of number operations are an example of a mathematical model in this context. The teacher is the expert who has knowledge of the mathematics represented by the model and the intention is to use the model to make the mathematics accessible to the children.

The idea of a model is used in a different way within the Realistic Mathematics Education (RME) approach. Here models emerge as individuals interact with particular activities (see Report No. 17, Chapter 5, Section: [*The First Approach: Working with Children's Thinking and Understanding \(RME\)*](#)). Gravemeijer and Stephan (2002, p. 148) say that (in the RME approach) 'modeling is seen as an organising activity from which the model emerges' and that 'subsequent acting with these models will help the students (re)invent the more formal mathematics that is aimed for'. An example of this is where the empty number-line can be used to model children's informal strategies for addition. A child might intuitively respond to a problem involving the sum of 26 and 18 by using a strategy such as $26 + 20 - 2$. For many children such intuitive methods are coherent with their emerging number sense. Base-ten blocks, the conventional materials used in many classrooms for

multi-digit addition, do not easily lend themselves to modeling these intuitive strategies. The number-line, as described here, supports children's strategies and encourages the development of increasingly sophisticated ones. In this case the model is being used to fit with, rather than to steer children's thinking. While at first children use the empty number-line as a model of their informal solution strategy (a model *of* a situation), gradually they become able to use the number-line for thinking about mathematical relations between numbers (a model *for* thinking about number relations). Proponents of the RME approach argue that working in this way, children develop deeper and more flexible understandings that can be applied to a range of situations.

English and her colleagues put a somewhat different emphasis on models and modeling (English, 2007; English & Sriraman, 2010). From their perspective, models are 'systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other familiar system' (English, 2007, p. 121). For them, modeling problems 'are realistically complex situations where the problem solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artefacts or conceptual tools that are needed for some purpose, or to accomplish some goal' (English & Sriraman, 2010, p. 173). Thus, from this perspective, mathematical modeling is an approach in which problem-solving is not separate from but integral to the understanding and development of new concepts. English and Sriraman (2010) regard mathematical modeling as an advance on the usual problem-solving that occurs in schools because:

- it often involves quantities or operations that go beyond those encountered in word problems
- it encourages children to mathematize as they try to make sense of a particular situation
- it uses contexts that draw on several disciplines
- it encourages the development of a model (e.g., graph or table) that can be applied to a range of situations
- it encourages social interaction and collaboration as students usually work in small groups or teams to solve the problems.

While much of their research on mathematical modeling in primary school concerns older children, English and Sriraman maintain that it has a rightful place in the very early years where important foundations for future learning of mathematics are laid. In particular, they propose the development of statistical reasoning through mathematical modeling and suggest as an example the pursuit of a question such as, 'Is our own playground fun and safe?'. In order to develop models to address this question, children engage in an iterative fashion in (i) refining questions and identifying attributes; (ii) measuring attributes and recording initial data; (iii) organising, analysing, interpreting, and representing their data; and (iv) developing data-based explanations, arguments, and inferences, and sharing these with their peers. Other questions might arise out of

this work such as, 'How can we make our playground safer?'. It seems to us that this perspective on modeling shares many emphases with those of the *Project Approach* which is discussed later in this chapter.

Thus, while their focus is on modeling as addressing realistically complex situations rather than modeling as an organising activity of more usual mathematics problems (see Gravemeijer & Stephan above), they align with the RME perspective in their emphasis on mathematization and on the development by the learner of a model that can be used in a variety of situations. We see that both have a significant role to play in a redeveloped mathematics curriculum for 3- to 8-year-olds. We suggest that this can best be conveyed in the curriculum presentation by the provision of detailed exemplars illustrating the two interpretations of modeling.

Cognitively Challenging Tasks

Stein, Grover, and Henningsen (1996) define a mathematical task as an activity 'the purpose of which is to focus students' attention on a particular mathematical idea' (p. 460). They maintain that the tasks used in classrooms are integral to the kinds of mathematical thinking in which students engage, and therefore to learning outcomes. They make particular reference to 'cognitively challenging tasks' as a means to promote higher-order thinking. Sullivan, Clarke, & Clarke (2013) argue that the overarching aim of mathematical proficiency implies using a variety of task types – an implication of this is that emphasis is not placed exclusively on worked examples that predominate textbook activities (and that are aligned with the development of procedural fluency). Account is also taken of rich and challenging activities that build on what children know mathematically and experientially, that allow them time and opportunities to make decisions, and that foster collaboration and communication. Drawing on a large body of research, Anthony and Walshaw (2007) draw the following conclusions about tasks:

- Open-ended tasks support student thinking and exploration. The openness relates to a range of 'correct' solutions and/or a range of ways to achieve one or more solutions.
- Tasks should provide students with opportunities for success, present an appropriate level of challenge and promote student agency and personal interest.
- When designing and implementing tasks, it is important that the goals and activities are responsive both to individual students' levels of understanding and to the discipline of mathematics.
- Differentiation can be facilitated by providing the same basic task to all students and taking individual needs into account (e.g., extra supports, extension activities etc.).
- Productive task engagement requires that tasks are closely linked to a student's current level of knowledge and understanding but are 'just beyond' his or her cognitive reach.

- In order to make tasks accessible, it is important that they are set in contexts that are 'realistic', that is, that allow learners to think in 'real' ways. The contexts can be real or imaginary settings that illustrate how mathematics is used. Some studies have found that the use of contexts can disadvantage children (particularly those in low-SES communities) who may be more literal in their interpretation of the problem situation (Cooper & Dunne, 2000). This does not mean abandoning realistic contexts but rather avoiding tasks that use mathematics to solve problems in unrealistic ways or those that use unrealistic or unfamiliar situations.
- Tasks can remain cognitively challenging throughout a lesson if emphasis is placed on ways of thinking rather than on correct procedures, if sufficient time is allocated to completion of the task and if there is a continued emphasis by the teacher on justification and explanation.

Formative Assessment

The NAEYC/NCTM (2002/10) position statement on early childhood mathematics states that, in providing high-quality mathematics education for young children, teachers and other professionals should 'support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies' (p. 9). The statement emphasises the importance of assessment when planning for ethnically, culturally, and linguistically diverse young children and for children with special needs. It also emphasises the use of assessment outcomes to plan and adapt teaching and curriculum. It notes that young children may invent their own mathematical ideas and strategies, which are quite different from those of adults, and that these need to be recognised. The NRC (2009) report, *Mathematics Learning in Early Childhood*, links the use of formative assessment (observation, tasks, interviews) to intentional or planned teaching, with assessment outcomes informing decisions about future learning.

In Report No. 17 ([Chapter 6](#)), we reviewed a range of formative assessment methods that can provide valuable information about young children's mathematical development, though it was stressed that it might be appropriate to use multiple methods on some occasions (e.g., an observation or task followed by an interview). The methods, which are consistent with the approach to assessment described in *Aistear* (NCCA, 2009a), include:

- *Observations* – structured observation of a child's engagement in mathematics. Learning stories (Carr, 2001) were identified as an approach to recording observations that could include a child's dispositions.
- *Tasks* – pre-designed or teacher-designed activities that provide insights into a child's mathematical understanding (e.g., Yelland & Kilderry, 2010).
- *Interviews* – focused conversations that explore in depth children's thinking and reasoning through questioning (and observation), generally about tasks that the child undertakes as part of the interview (e.g., NRC, 2009).

- *Conversations* – frequent but less-detailed questioning about a child’s mathematical thinking, arising in the course of completing tasks or other activities.
- *Pedagogical documentation* – dialogue and reflection by teacher and child on a range of artefacts (e.g., pictures, recordings, work samples) that arise from engagement in mathematical tasks (MacDonald, 2007).

The research literature notes that relatively few studies have demonstrated clear links between assessment outcomes, planned instruction, and growth in children’s mathematical learning (e.g., NRC, 2009), but that learning paths are a framework that teachers can draw on for the assessment of children’s learning (see Report No. 17, Chapter 6, Sections: [Interviews](#); [Conversations](#); [Supporting Children’s Progression with Formative Assessment](#)).

Formative assessment was highlighted in Report No. 17 as being most consistent with sociocultural, child-centred approaches to mathematics education, and the unsuitability of more summative assessment measures for use with young children was noted. On occasion, formative assessment information can be complemented by information derived from screening or diagnostic tests.

Practices in Integrative Contexts

Good pedagogy is enacted in the course of everyday activities in early education settings, and it is characterised by the features of good mathematics pedagogy identified in Chapter 1 (see [Table 1.1](#)). Pedagogical practices discussed in this section focus on engaging children in play, in story/picture-book reading, in project work, and on mathematics learning through arts or physical education. These provide some important contexts in which young children in early educational settings engage with mathematical ideas. Other contexts, for example, problem-solving in specific content areas of mathematics, are considered in Chapter 3. Moreover, there are opportunities for mathematics development across all areas of learning, and not just the ones discussed in this chapter. For example, spatial concepts and spatial relations can be developed through exploring a geographically-focused theme.

The practices highlighted here promote children’s use of a range of tools, including digital tools. The learning activities arise from children’s interests, concerns, and questions and the educator links these to learning goals. The practices are generally holistic in nature and facilitate an integrated approach to mathematics education for children aged 3–8 years. However, a clear focus on mathematical goals is required, even within an integrated approach. As emphasised earlier in the introduction of this chapter, it is essential that the meta-practices discussed above (the promotion of math talk, development of a positive disposition, emphasis on mathematical modeling, use of cognitively challenging tasks, formative assessment) permeate all learning activities if children are to develop mathematical proficiency. The relationships among these elements are illustrated in Figure 2.1.

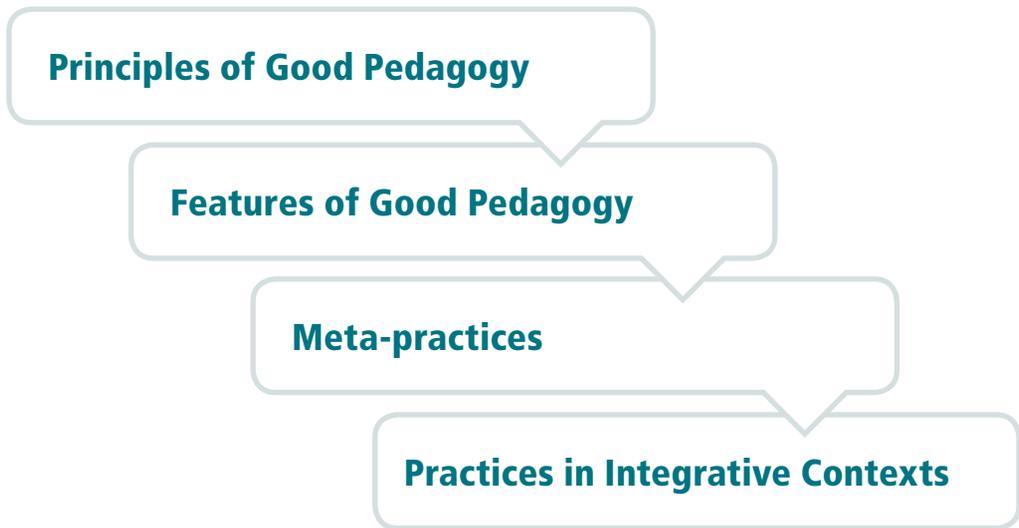


Figure 2.1: Relationships between Different Aspects of Pedagogy

Play

Given the importance of play as a learning process for young children, it is essential that good mathematics pedagogy recognises this fact, honours it and harnesses its power. Sarama and Clements (2009) identify three types of play in which children engage with mathematics: sensorimotor play, symbolic or pretend play, and games with rules. *Aistear* (NCCA, 2009a) promotes a range of different types of play, i.e., ‘creative’, ‘games with rules’, ‘language’, ‘physical’ and ‘pretend’. Although not outlined specifically in *Aistear*, all of the above types of play contribute in their own way to children’s mathematical learning and can offer valuable opportunities for playful mathematical experiences (Ginsburg et al., 2006; Perry & Dockett, 2008).

The various types of play strengthen children’s mathematical learning and understanding in different ways. The following examples highlight ways in which mathematical skills and concepts can be developed in early years settings, in both indoor and outdoor environments:

- Physical play refers to physical, exploratory, manipulative and constructive play. It is the most common type of play in very young children (Montague-Smith & Price, 2012) as it involves bodily movements such as clapping, hopping and jumping. Through engaging in physical play experiences, children can learn a variety of mathematical concepts and skills. Physical play experiences include participating in games and activities that develop the vocabulary of position and movement; identifying and comparing shapes and patterns within the environment;

exploring and manipulating materials and identifying their characteristics; and comparing sizes of objects and counting them. Through engaging in constructive play children develop mathematical skills such as problem-solving, visualisation, spatial awareness and reasoning, tessellation and pattern-making.

- Pretend play encompasses make-believe, dramatic, socio-dramatic, role, fantasy and small world play. Pretend play involves children being creative and using their imaginations with objects, actions and in role-playing. Through participating in pretend play, children develop early literacy and numeracy skills. Through playing with real objects they develop mathematical skills and engage with concepts such as number operations related to counting, calculating, problem-solving, number, measure and time. Using objects to symbolise other things, children move from thinking in the concrete to thinking in the abstract (NCCA, 2009a).
- Creative play involves children exploring actions and materials and communicating their ideas. Through creative play children develop a variety of mathematical skills in meaningful contexts. For instance, children playing with junk and recycled materials can make models, explore the properties and characteristics of 2-D and 3-D shapes, investigate symmetry and tessellation and develop mathematical reasoning and problem-solving by constructing and deconstructing shapes.
- Language play involves children playing with sounds and words. Children learn mathematical language through discussion in playful situations, e.g., shopping, cooking or number stories. When children engage in play they can use objects to symbolise or create something new and, in doing so, can use mathematical language associated with the new object. Through counting concrete materials in playful contexts number language can be extended.
- Games with rules include activities where children follow a specific set of instructions or negotiate their own rules. Games with rules provide opportunities for collaborative learning and for the development of mathematical activities including reasoning, problem-solving, classifying and ordering. These activities can include people games with children following directions such as 'Simon Says', games measuring time such as 'What time is it Mr Wolf', movement games and number and board games. For example, in 'Simon Says' children might be asked to clap three times, or take two steps then one step, altogether three steps. Accommodations should be made for language levels. In invented games children can select appropriate manipulatives to support their learning e.g., dice, playing cards and number cards.

The playful activities above contribute to the development of aspects of mathematical proficiency such as conceptual understanding and productive disposition. They also present valuable opportunities for observation and assessment of mathematical understanding and learning.

These play activities listed above are, for the most part, teacher-initiated and directed. When planning for mathematical development through playful activities, educators need to be also

mindful of the fact that child-led play offers rich opportunities for mathematical learning and understanding. *Aistear* (NCCA, 2009a, p. 53) stresses that children love to make choices about when, what, where, how and with whom to play. Educators should ensure that quality resources are available so that as they play, young children can construct and reinforce mathematical knowledge. Through engaging with these quality resources children can, for example, construct a model, identify numbers in the play environment, exchange coins for goods, find a block to fill a space and choose blocks to copy a sequence or a pattern.

Despite the strengths of play as outlined above, it is also recognised that not all playful activities lead to mathematical understanding (Ginsburg, 2006). Research indicates that children do not always engage in mathematical learning opportunities as play can often be restricted by such contextual factors as lack of resources, curriculum overload, limited space, and class or group size (Kernan, 2007; McGrath, 2010). Children's dispositions, arising from their experiences, might also be implicated here (e.g., Dunphy, 2006), and some children may need encouragement and support to engage in a mathematical way in play, or in mathematical play. Another limiting factor may be the undervaluing of the potential of play by adults, who may be under pressure to provide evidence of specific types of learning (Wood & Attfield, 2005). These limitations also present pedagogical challenges for the educator who attempts to implement a play-based mathematics curriculum.

Despite the challenges outlined above, it is evident that play is a key process through which children learn mathematics and practitioners can overcome many of the challenges in ensuring that optimum outcomes for children by careful and resourceful planning. Observing children at play, thinking creatively about play spaces and resources both indoor and outdoor, participating and interacting in playful situations, co-constructing with children and assessing the effectiveness of play experiences are all aspects of pedagogy which are essential to productive and worthwhile mathematical play-based experiences for children (e.g., Kernan, 2007).

Story/Picture-Book Reading

Picture-Books

Research indicates clearly that children's literature contributes greatly to the process by which young children acquire mathematical thinking. It does so by offering enjoyable and meaningful contexts – paper-based or digital – in which mathematical content and concepts may be explored and developed (Casey, Kersh, & Young, 2004; Hong, 1999; van den Heuvel-Panhuizen, 2012). Literature for young children generally includes pictures since artwork is an important feature in the education of pre-literate children. In most story books the illustrations, as well as the text, play a prominent role in the telling of the narrative and the creation of meaning (Elia, van den Heuvel-Panhuizen, & Georgiou, 2010) so these books are generally referred to as 'picture-books' (van den Heuvel-Panhuizen & Elia, 2012). Picture-books usually show mathematical

concepts visually (Murphy, 1999) and therefore support children's understanding of abstract concepts (Montague-Smith & Price, 2012). Through engagement with picture-books, young children are presented with rich contexts in which they encounter problematic situations, ask questions, reason mathematically and have conversations with adults and peers, all of which can lead to the use of mathematics-related language (Anderson, Anderson, & Shapiro, 2005; Hong, 1996; van den Heuvel-Panhuizen, 2012; Young, 2001).

A number of studies have examined how the use of picture-books enhances young children's mathematical understanding. Hong (1996) investigated the impact of a programme that focused on mathematics-related storybook reading, discussion, follow-up activities and play on children's performance in specific mathematically-rich tasks. Her findings revealed that the 4- to 6-year-old children involved in the study did significantly better on tasks involving classification, number combinations and shape compared to the control group. It was suggested also that the experimental group were more favourably disposed towards mathematical learning and thinking, and chose to spend more time engaging in mathematical tasks and in the mathematics corner.

Research by Young-Loveridge (2004) indicated that 5-year-old children involved in an intervention programme which focused on listening to number stories and rhymes, as well as playing number games scaffolded by adults, demonstrated significant improvements in numeracy skills when compared with a group who were not involved in the programme. An important aspect of this programme was the fun the children had engaging in activities. This illustrates the role that parents have in supporting and enhancing the mathematical abilities of children, especially during the transition to school.

Interactions during Story/Picture-Book Reading

Casey et al. (2004) endeavoured to embed mathematics in a story context through the use of six problem-solving adventure stories. The texts were designed to develop the children's spatial and analytical skills. Their results indicate that the children who encountered geometry within a storytelling context using one of the above books achieved greater success in their mathematical activities with blocks than those who did not engage with similar content within a story context. In a follow-up study using one of these picture-books, Casey, Andrews, Schindler, Kersh et al. (2008) investigated the use of block-building interventions to develop children's spatial-reasoning skills. A puppet was used as the story-teller and his presence provided a meaningful context for the carrying out of mathematically-related tasks. Another study showed that girls benefitted more from geometry interventions than boys (Casey, Erkut, Cedar, & Young, 2008). This is an important finding given that we know that girls, regardless of age, are at a disadvantage in solving spatial problems compared with boys (e.g., Casey, 2009). However, it may be the quality and depth of the spatial language environment experienced by girls, rather than exposure to specific spatial activities, that are more critical for girls' early acquisition of spatial skills (Dearing et al., 2012). It therefore appears that story-book reading combined with play, activity and focused language development provide for optimal learning.

Children can be mathematically engaged by listening to a picture-book being read aloud, even without additional teacher intervention (van den Heuvel-Panhuizen & van den Boogaard, 2008). In this study an analysis of children's spontaneous mathematically-related speech during a story-reading session – using a book not designed specifically to teach mathematics – indicated that they used both spatial orientation-related utterances and number-related utterances. In a study by Elia et al. (2010), a picture-book that was written specifically for the purpose of teaching mathematics was used. As in the previous study, the teacher did not give explicit instruction or question the children as she read the story. The findings again revealed that the children used mathematics-related speech. This was thought to be due to the fact that they were presented with a context that made sense to them.

However, this is not to underestimate the effect of appropriate teacher intervention. For example, Elia et al. also reported that pictures with a representational content were found to elicit mathematical thinking to a greater extent than pictures that included informational functions. Therefore, while pictures in stories may be perceived as valuable tools in enhancing children's mathematical thinking, there needs to be adult interaction if children are to benefit fully from the mathematical-informational purposes of a story. Björklund and Pramling-Samuelsson (2012) stress that even though mathematical concepts may be embedded within a story context, many children cannot recognise these. They suggest that the three important factors that must be borne in mind when storyreading are: shared attention, reasoning and meaning, and the teaching of specific mathematical content (all of which have the potential to contribute to the development of mathematical proficiency). In the case of the latter, they argue that the teacher must have an intended mathematical objective when reading a story if the children are to achieve optimum learning from the story context. Indeed, van den Heuvel-Panhuizen and Elia (2012) stress that the reading style that best suits the power of the picture-book to develop children's mathematical thinking and understanding is dialogic book reading (e.g., Cunningham & Zibulsky, 2011). Here the emphasis is on letting the picture-book provide the context for the co-construction of meaning between child and adult, with the balance of power in favour of the child.

Pramling & Pramling-Samuelsson (2008) carried out a study using a storytelling context where children were required to solve a mathematical problem and to represent their answers through illustrations. A key finding of the study is the need to make explicit to children different ways of representing mathematical information. The authors caution against the inclusion of extra resources by teachers, which in this case were pictures, in an effort to support young children's problem-solving. Rather than supporting the children's learning, the pictures created uncertainty as the children were attracted to the incidental rather than the critical in the story.

Selecting Books

As the above studies indicate, picture-books vary in the amount and types of mathematical knowledge they present. Thatcher (2001) identifies criteria for selecting books for teaching mathematics, and offers advice on the effective use of literature in the teaching of mathematics to young children. In a more recent study, van den Heuvel-Panhuizen and Elia (2012) draw on extensive research literature to examine basic issues in relation to the characteristics of picture-books that support young children's mathematical understanding. They used recent findings and theory to produce a framework of learning-supportive characteristics of picture-books for learning mathematics. Their framework, presented below, should prove useful to those who wish to evaluate the suitability of certain picture-books for young children's mathematical development (van den Heuvel-Panhuizen & Elia, 2012, p. 34).



Figure 2.2: Framework of learning-supportive characteristics of picture-books for learning mathematics

Experiences with mathematically-related stories have the potential to promote aspects of mathematical proficiency, including procedural fluency, adaptive reasoning and a productive disposition.

Project Work

The *Project Approach* is recognised as offering opportunities for mathematical development. The term 'project' refers to an in-depth study of a particular topic undertaken by small groups of children (Katz & Chard, 2000). It is designed to assist young children to make deeper and fuller sense of events and experiences and to support their learning by encouraging them to make decisions and choices in collaboration with their peers and teachers (Katz, 1998). Children's interests provide the stimulus for the topic or project to be investigated. The *Project Approach* presents children with opportunities to make sense of real-life problems (NAEYC/NCTM, 2002/2010) as most projects involve a wide variety of types of problem-solving (Helm & Katz, 2010). Children's mathematical concepts and language may be developed across subject matter boundaries (NAEYC/NCTM, 2002/2010).

Projects involve children in investigating a topic of interest or importance to them. The impetus for the project comes from the children themselves. A key feature of a project is that it is an investigation that encourages the active participation of children in the planning, development and assessment of their own work (Katz & Chard, 2000). The essence of the *Project Approach* is to engage children in a complex and interesting project that exploits and elaborates on the mathematics that arise in the course of the activity (Ginsburg & Golbeck, 2004).

The roots of the *Project Approach* can be traced to the work of Dewey and Kilpatrick (Hall et al., 2010). In Reggio Emilia schools, the word *project* has a broader meaning in that it involves investigation and expression by means of various symbolic languages (Tarini, 1997). Project work has been described and advocated in the *Primary School Curriculum* (Government of Ireland, 1999a) and *Aistear* (NCCA, 2009a). The *Primary School Curriculum* emphasises that for young children the distinctions between subjects are not relevant. It stresses the importance of a coherent learning process where connections are made between learning in different subjects. The theme of *Exploring and Thinking* in *Aistear* (NCCA, 2009a) focuses on children making sense of their environment. *Aistear* highlights the importance of the role of the adult in project work. The adult enhances the children's learning experiences during the project process by providing resources, participating in project-related activities and interacting with children. The adult showcases the children's projects (by displaying photos or showing video) and helps them share their work with other children and parents.

The Project Approach in Action

Boaler (1997) asserts that the *Project Approach* enhances children's problem-solving skills as they are consistently challenged to solve mathematical problems that occur as the project unfolds. According to Helm and Katz (2010), the *Project Approach* helps children to

- generate an awareness of the function of number and quantity concepts
- create a reason to quantify information
- represent quantities with numerals
- see reasons to classify and sort
- develop categories
- use tools for investigation, experimentation and observation
- compare and order objects
- engage in mathematical thinking
- use measurement, counting and graphing
- develop an awareness of shape, area, distance and volume
- construct models, drawing diagrams and charts and creating play environments.

The *Project Approach* weaves mathematics with young children's everyday experiences in the early education setting and offers rich opportunities for the development of mathematical thinking and understanding. Of particular relevance is the incorporation of digital tools in young children's projects. As discussed in Report No. 17, using technology is an increasingly important avenue of learning and expression for children. For example, Kalas (2010) describes children's engagement with technology and digital tools as they pursued a project on *My Town*. As the project developed, children explored direction and location using techno-toys on their specially-constructed floor map. In doing so, they explored spatial concepts and developed the language of spatial relations (e.g. beside, towards). As they investigated long/short and longest/shortest routes, there were opportunities to develop algorithmic thinking (processes or rules for calculating). Children also documented their peers using a video-camera, and this was used as a basis of discussion, explaining, reasoning and justification. This project can be seen to provide rich opportunities to develop a number of aspects of mathematical proficiency. Further examples of rich mathematical activities or investigations drawn from the literature are outlined below.

A study of water (adapted from Dixon, 2001)

Children form groups that focus on different aspects of water e.g., 'What can water do?'; 'Where does water come from?'. Children decide on suitable activities and experiments and conduct these within their groups, for example, experiments relating to capacity and sinking and floating. Children document processes through diagrams, drawings, charts, photographs, data and models. Children demonstrate activities to the rest of the group and, in doing so, explain mathematical processes.

Making apple sauce (adapted from Ginsburg & Golbeck, 2004)

Children decide how many jars of apple sauce are required; they count the number of jars; they 'read' a pictorial recipe for apple sauce; they discuss ingredients to purchase; they walk to the supermarket and discuss the route; they weigh ingredients; they compare size, shape, colour and price of fruits; they exchange money for apples and calculate change and, on returning to the school, they make the apple sauce which leads to further investigations.

The pizza project (adapted from Gallick & Lee, 2009)

Children discuss the topic of food, recipes for pizza and the development of a 'topic web' based on these. They also sequence the making of a pizza; estimate, measure and cut circles of paper to represent pizza slices; develop a pizza-themed play area; order and pay for pizza; and share pizza amongst friends. From a teaching and learning perspective, projects are a valuable approach to organising mathematical activities for young children (Katz & Chard, 2000; Ginsburg & Golbeck, 2004). While some learning experiences may look like projects, a learning experience cannot be considered to be a project unless the elements of child initiation, child decision-making and child enjoyment are present (Helm & Katz, 2010). It can be seen that project work, carefully implemented, can develop each of the strands of mathematical proficiency.

Learning Mathematics through the Arts and Physical Education

This section examines specific ways in which links can be established between the arts (music, the visual arts, drama) and mathematics and sets out options for how the mathematics curriculum for 3- to 8-year-olds might seek to strengthen links between mathematics and other areas of learning (and vice versa).

Music

Music is a rich context in which educators can develop children's mathematical language and concepts. Shilling (2002) suggests that, through a classification of sounds and movement, children's mathematical understanding and skills are enhanced. She identifies a strong link between the order, timing, beat and rhythm of music and attributes of mathematics such as counting, sequencing and understanding time and order. The integration of music into children's mathematical and physical activities supports their logical and rhythmic development and enables teachers to make learning both music and mathematics more meaningful for the children (Kim, 1999; McGrath, 2010; Montague-Smith & Price, 2012; Pound, 1999; Shilling, 2002). Engaging children in making and responding to music may also contribute to the development of other skills and attitudes that are important for mathematics such as concentration, creativity, perseverance, self-confidence, and sensitivity towards others (Fox & Surtees, 2010).

Among the ways in which music and mathematics are linked are the following:

- Young children come to school with intuitive knowledge of musical patterns and rhythms (Shilling, 2002). Their first musical experiences can often include lullabies, nursery rhymes, stories and songs. Teachers can create mathematical opportunities for children to respond to the rhythms, patterns and sequences embedded in music.
- Children can learn and practice counting through the recitation of rhymes, chants and songs that have counting-related words (Kim, 1999).
- The development of number sequences is achieved through repeated rhymes, songs and stories. Children move on to associate a value to these number names (McGrath, 2010; Montague-Smith & Price, 2012; Pound, 1999).
- In making and responding to music, children should have an opportunity to create a range of musical patterns and to understand such musical elements as pitch (gradations of high/low), dynamics (gradations of volume, louder/quieter, silence), tempo (different speeds) and structure (the ways different sounds are organised) (Fox & Surtees, 2010). Fox and Surtees show how teachers can specify particular mathematics objectives as they plan lessons or projects involving music. They also suggest that teachers consider how aspects of mathematics can be assessed in cross-curricular contexts such as music.

Visual Arts

Pattern and shape are key features of both the visual arts and mathematics. In the visual arts, children encounter colour, form, texture, pattern and rhythm, and shape (Government of Ireland, 1999c). In mathematics, they discover patterns of number and shape, symmetry, tessellation, and the properties of a range of 2-D and 3-D shapes.

A key aim of the visual arts curriculum is 'to develop the child's awareness of, sensitivity to and enjoyment of visual, aural, tactile and spatial environments' (Government of Ireland, 1999c, p. 4), while awareness of the visual and spatial qualities in the environment is also important for mathematical understanding, and for enhancing children's ability to apply mathematical knowledge in the environment (i.e., in real life). Although the current visual arts curriculum provides specific suggestions for linkages with other areas of the curriculum, just a few of these relate specifically to mathematics. The following are some ways in which mathematics might be integrated into the visual arts:

- identifying 2-D shapes (circles, triangles, rectangles, squares) in fabrics
- repeating patterns, translation and rotation
- measuring fabric samples and investigating perimeters
- looking and responding: identifying and talking about geometric patterns and symmetry in pictures

- identifying light and dark areas
- using ICT to design and discuss the properties of a print.

With the youngest children, play with malleable materials such as clay or dough is rich in opportunities to experience the way in which a given quantity can change shape. The educator can use the opportunities presented to help children to understand the meaning of, for example, measure words such as *long* and *short*. Children can be supported to use and refine key vocabulary, as appropriate, to include words such as *longer than* (comparatives) and *longest* (superlatives). Activities such as printing allow children to begin to develop concepts of area and perimeter, and in this context they may also make connections with patterning as they experiment with sequencing elements and/or groups of elements and repeating sequences to form patterns.

Drawings and mark making can be used by children to convey their growing awareness of number and quantity. The educator, in considering the child's verbal explanations of the graphics he/she creates, can gain insight into the child's current and developing understandings of the ways in which we use mathematical language and record this by making marks. Through discussion, the child also develops abilities to translate mathematics from one language (verbal) to another (graphic) (Worthington & Carruthers, 2003).

Again, Fox and Surtees (2010) argue that integration is most effective when specific learning outcomes are identified and consideration is given to how achievement of the objectives can be assessed. They also point out that the arts provide a forum in which children's confusions about particular aspects of mathematics can be addressed (e.g., the base of 2- and 3-D shapes does not need to be parallel to the bottom of the page – orientation can vary; length is not necessarily longer than width).

Drama and PE

Role-play offers many opportunities for children to engage with mathematical concepts and skills. Story contexts such as 'The Three Little Pigs' can give rise to a range of mathematically-related play, especially if appropriate props are provided to stimulate mathematical thinking (e.g. Pound, 2008). The educator can develop concepts through discussion as appropriate. For example, in many role-play contexts children can be challenged to consider questions about quantity. Phrases such as 'just enough' (equality), 'not enough' (less than) and 'too many' (greater than) can be used and their meaning explored in the context of the play.

Although the current curricula in drama and physical education do not emphasise specific approaches to integrating these areas with mathematics, they give rise to authentic contexts that can be used to develop children's understanding of mathematics, for example:

- Games which involve throwing beanbags into a hoop, bouncing a large ball, skipping and then counting to answer the question 'How many?'

- Forming groups for games, representing basic processes such as addition or subtraction, by combining or separating groups of children. Partitioning of numbers can be explored – for example, a group of 7 children could explore the different ways in which 7 could be partitioned by splitting into two subgroups ($6 + 1$; $5 + 2$ etc.).
- Creating 2-D shapes such as triangles or rectangles using children’s bodies, and discussing the properties of such shapes.
- Visualising the properties of 3-D shapes such as cylinders, cuboids and triangular prisms by pretending to reside inside a shape and describing the sides, angles and corners and showing how to travel inside the different shapes.
- Examining and discussing the movement involved in dance to identify lines, shapes, pattern and symmetry.
- Participating in swimming or athletics and calculating times and distances. Very young children can be exposed to mathematical vocabulary through everyday discourses such as swimming lessons (Jorgensen & Grootenboer, 2011 as reported in MacDonald, Davies, Dockett, & Perry, 2012).
- Engaging in problem-solving activity in role-play.

Clearly, much can be gained from linking aspects of the arts and PE curriculum with mathematics. Key issues for curriculum development include the following:

- Should curricula in the arts and PE specify in more detail the connections with mathematics that can be made, by, for example, identifying learning outcomes in mathematics that can be achieved through activities in the arts?
- Should the mathematics curriculum include learning outcomes that relate specifically to application of mathematics in other curriculum areas?
- Should teachers be expected to assess children’s ability to integrate mathematics into other subject areas? Should this be done as part of assessing mathematics? Or separately?

Digital Tools

In Report No. 17, we gave some attention to the role of tools in the construction of mathematical knowledge. We discussed how, from a sociocultural perspective, tools – including both physical artefacts and symbolic resources – are an integral aspect of human cognition and activity. Cultural tools are considered to influence the ways in which people interact with and think about the world (see Report No. 17, Chapter 2, Section: [Sociocultural Perspectives](#)). The physical artefacts include manipulative materials, pens, books and computers, while symbolic resources include language, drawings and diagrams (Armstrong et al., 2005). Elsewhere in this report we give consideration to tools such as language (for example, the section on [Promotion of Math Talk](#) in this chapter) and concrete materials and drawings (for example, the section on [Representing](#), Chapter 3).

Digital tools deserve particular attention because of their key role in children's lives. However, schools and teachers are generally not systematically incorporating curriculum or NCCA guidelines on digitally-related activity in mathematics into the work that takes place every day in classrooms (DES, 2010, p.12). Furthermore, children in junior classes experience a narrower range of digitally-related activity than children in senior classes (DES, 2008). Beyond Ireland, many researchers (Perry & Dockett, 2004; Perry, Lowrie, Logan, McDonnell, & Greenlees, 2012) acknowledge that there is a dearth of evidence-based research investigating young children's use of technology in mathematics learning. The available research tends to focus on screen-based technologies, calculator use or the role of the teacher (Fox, 2007; Mulligan & Vergnaud, 2006; Perry & Dockett, 2007b; Yelland, 2005). However, some researchers have begun to examine the potential of computer-based tools for mathematical representation by young children (Clements & Sarama, 2007; Highfield & Mulligan, 2007; Moyer, Niezgodna, & Stanley, 2005).

Digital Technologies as Learning Tools

'Children born...[today] are growing up in a world in which digital technologies are not only widely accessible to most families living in Western societies, but so commonplace as to be unremarkable' (Plowman, Stephen, & McPake, 2010, p. 135). Our classroom environments need to reflect this ubiquitous presence so that young children can play with and experience these digital tools that have cultural significance in order to gain a sense of empowerment and control over the technology (Price, 2009). Research has demonstrated that the use of these tools has the potential to significantly improve educational opportunities for young children (Price, 2009; Siraj-Blatchford & Whitebread, 2003) and can benefit young children's learning in a range of ways (Downes, 2002; Clements & Sarama, 2003; Haugland & Wright, 1997; Plowman & Stephen, 2005; Yelland, 2005, 2007; Zevenbergen & Logan, 2008).

If these tools are used as 'an object to think with' (Papert, 1980) or a 'mindtool' (Jonassen & Carr, 2000), young children can develop higher-order thinking and engage in knowledge construction. These tools enable children to revisit and reflect on their prior learning, so that they can become more actively engaged in the learning processes (Bauer & Kenton, 2005). This reflection 'allows further learning to be sequentially linked and re-constructed in the light of previous thinking' (Highfield, 2010a, p. 181). Digital tools thus have the potential to assist in the development of children's mathematical proficiency, particularly in relation to the strands of adaptive reasoning, strategic competence, and productive disposition. However, a major challenge facing early childhood educators is to begin thinking about digital technologies as learning tools which children learn 'with' and not 'from' (Jonassen, Howland, Moore, & Marra, 2003).

Research indicates that young children's technology play has been one of the most contentious issues faced by early childhood education in recent decades. At the centre of this debate have been ideas of developmental appropriateness and fears that technology use creates risks for social and emotional development (Cordes & Miller, 2000; Healy, 1998; Highfield, 2010a). Some fear that

communication might be inhibited by technology play but this has been challenged by many researchers (e.g., Kelly & Schorger, 2001; Hyun & Davis, 2005). Indeed, Clements and Sarama (2004) take the opposite position, stating that 'computers are catalysts of social interaction' (p. 341).

Many of these fears stem from a restricted view of technology (e.g., a focus on desktop computers), which in turn can lead to a restricted view of play. Yelland (2010) calls for a re-conceptualisation of play to incorporate activities using new media as playful experiences that are supported by adults. She argues that exploration in virtual worlds requires us to rethink the nature of play. This contemporary view of play incorporates new technologies that afford opportunities for young children to play and communicate in multiple modes so they are able to acquire deeper understandings about how things work and connect and are relevant to their lives. Arising from the literature, we offer some representative examples of the use of digital technologies for supporting early mathematical development. Since much of the research is based on specific tools and software, these feature in the examples below.

Example 1: Techno-Toys

Technology has enabled the creation of a new generation of techno-toys that differ from traditional toys as they have embedded electronics, response systems and microchips that enable them to respond to children in some way. They can be categorised by their technical features or by their 'affordances' (functions and engagements that a toy may enable) which can be intended or unintended, as well as 'open-ended' (allows users to engage in child-controlled creative processes, e.g., *BeeBots*) or 'closed' (only allows user to respond in limited ways). Highfield (2010a) developed a classification system which incorporates potential possibilities afforded by techno-toys. These include opportunities for young children to: represent and create; manipulate; program; communicate; investigate; simulate and model; problem-solve and think strategically; and play a rules-based game. Highfield (2010b) outlines five scenarios of children's digital play with techno-toys. The activities in which the children engaged included patterning, number and numeric structure, spatial awareness and positional language, size, ratio and proportion, and time. In one of the scenarios, Highfield describes the use of *BeeBots*, a techno-toy that has potential to support the mathematical development of children aged 3–8 years.

BeeBots are simple robotic toys whose movements can be programmed by children. This affordance of programming the *BeeBot* resonates with the research on Logo which indicated Logo's usefulness as a tool in teaching and learning mathematics (Butler & Close 1989, 1990; Clements & Sarama, 1997; Hoyles, 1987; Hoyles & Noss, 1992; Yelland, 1995), particularly in relation to the development of geometry and spatial concepts (Clements & Battista, 1992). By interacting and playing with the *BeeBots*, children are developing mathematical ideas (e.g., spatial awareness, positional language, ideas of directionality, concepts of measurement, estimation, counting, and transformational actions, including linear movement and rotation) and metacognitive processes (planning, problem-solving and reflection) (Highfield & Mulligan, 2008; Highfield, 2010b).

It has also been suggested that techno-toys such as the *BeeBots* have the potential to advance progress along learning paths by exposing children to concepts that they normally would not be

introduced to until a later age. However, it must be realised that exposure to advanced ideas does not ensure that children will grasp and understand these ideas or concepts as they may just 'wash over' the children without application or understanding (Highfield, 2010a).

Example 2: Software

Virtual manipulatives (e.g., pattern blocks, base-ten blocks, geo-boards, tangrams etc.) are 'an interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge' (Moyer et al., 2002, p. 373). They provide access to unlimited quantities of materials and can be used to help children develop ideas of composition and decomposition of number as well as patterns and relationships (Reimer & Moyer, 2005; Moyer et al., 2005). Virtual or digital manipulatives can draw on children's intuitions about physical objects and extend those objects to allow a new range of concepts, which were previously viewed as too advanced to be explored (Resnick et al., 1998). The use of virtual manipulatives has also been examined by Swan and Marshall (2010), who confirm the findings of an older study (Perry & Howard, 1997), concluding that 'their use did not guarantee success: the major benefit of the manipulatives comes from the discussion that goes on around them and explicit linking by the teacher to the mathematics they represent' (Perry et al., 2012, p. 177).

Teachers need to be able to capitalise on the sometimes unintended affordances of software. One example of this is a drawing package called Kidpix (<http://www.broderbund.com>), which is dynamic interactive software. With this, simple patterning can be collaboratively explored, as the tool's primary affordance is in creating and representing, while the secondary affordance is manoeuvring and manipulating images around the screen. Highfield (2010a) argues that traditional drawing techniques do not support such dynamic interaction and thus Kidpix may offer new learning opportunities for children. The software provides opportunities for geometric actions such as flips, rotations, shearing and scaling (Highfield & Mulligan, 2008, p. 19). As with all computer software, children can save their work, adding to it and changing it as they wish, so that future learning can be informed easily by prior experiences. Teachers can also use freely accessible tools to enable children to express their understanding of particular concepts. For example, using AutoCollage, children can easily construct a montage from the images they capture using the digital camera to illustrate shapes they see in the environment, e.g., sets of different numeric value, patterns observed in nature, insects with a set number of legs, etc.

Example 3: Interactive White Board

The interactive white board (IWB) has become a popular tool in Irish primary classrooms over the last five years in particular. However, in many settings, IWBs tend to be used by teachers predominantly as a replacement for the traditional blackboard rather than capitalising on its interactive possibilities for children's learning. There appears very little research on its efficacy in the context of early learning of mathematics. While a small number of studies look at the use of the interactive white board for supporting young children's mathematical development (e.g., Goodwin,

2008), much work needs to be done in this area. Given that the software which supports the use of the interactive white board often includes a bank of virtual manipulatives, observations made above about the use of virtual manipulatives should also be borne in mind when using the interactive white board in classroom settings.

In summary, there needs to be a concentrated effort for further research to move beyond just screen-based tools and examine how the full range of existing and emerging digital tools and computational devices can make powerful mathematical ideas accessible to and impact on young children's mathematical and meta-cognitive processes. However, this research also needs to be mindful of the learning environment and the complex role of the teacher for, as Clements (2002) points out, 'the curriculum in which computer programs are embedded, and the teacher who chooses, uses, and infuses these programs, are essential elements in realising the full potential of technology' (p. 174).

Conclusion

In Report No. 17, math talk, disposition, modeling, tasks and assessment all emerged as important factors in the theoretical discussions about mathematics education. In this chapter we surveyed the literature which offers a range of perspectives, and advice, as to the issues for educators in incorporating these elements into their practices. We saw that good mathematics pedagogy can be enacted when educators engage children in a variety of activities which have the potential to develop mathematical understanding. The activities should arise from children's interests, questions, concerns and everyday experiences. They may be generated across different areas of learning and they may utilise a range of tools, including digital tools. The potential of these activities for developing mathematical proficiency can best be realised when educators focus on children's mathematical sense-making.

The key messages arising from this chapter are as follows:

- Good mathematics pedagogy incorporates a number of meta-practices including the promotion of math talk, the development of a productive disposition, an emphasis on mathematical modeling, the use of cognitively challenging tasks, and formative assessment. A pedagogy incorporating these meta-practices supports the vision of 'mathematics for all'.
- A deep understanding of the features of good mathematics pedagogy should inform the ways in which educators engage children in mathematics across all areas of learning.
- Educators need to maximise the opportunities afforded by a range of tools, including digital tools, to mediate learning.
- Practices which reflect the features of good pedagogy contribute to the development of the strands of mathematical proficiency.

CHAPTER 3

Curriculum Development



Remillard (2005) maintains that the designers of curriculum materials must take account of the teacher-curriculum relationship and the underlying messages that their materials communicate to educators. In this chapter, attention is given to the overarching idea of mathematical proficiency as an aim of mathematics education for 3- to 8-year-old children, the processes that need to be developed in line with this aim and the content domains that need to be included in a redevelopment of the curriculum. While we recognise that the current PSMC (Government of Ireland, 1999a) encapsulates some of these ideas (e.g., inclusion of process skills), we argue for a reformulation of the curriculum which foregrounds mathematical proficiency as the main aim. Mathematization should be a key focus and its associated processes should be clearly indicated. We also propose a rebalancing of the focus on processes compared with content. Towards the end of this chapter, various ways in which learning paths might be used in formulating the mathematics curriculum are explored.

Although broad aims, process skills and content objectives for the teaching of mathematics are provided in the Irish PSMC (Government of Ireland, 1999a) (a teacher's guide with practical teaching examples is also available – Government of Ireland, 1999b), there remains an overly strong focus on the strand of Number and on implementation of procedures and textbook activities in most primary classrooms (see Report No. 17, Introduction, Section: *Performance Context*; this report, *Introduction*). The reasons for this are complex and manifold. However, a matter in need of attention is the lack of synchronisation between the aims, processes and content objectives. For example, problem-solving is highlighted in the introduction to the curriculum:

Developing the ability to solve problems is an important factor in the study of mathematics. Problem-solving also provides a context in which concepts and skills can be learned and in which discussion and co-operative working may be practised. Moreover, problem-solving is a major means of developing higher-order thinking skills...(Government of Ireland, 1999a, p. 8)

However, in the listing of content objectives, the solution and completion of practical problems are usually placed at the end of the sequence of objectives relating to strand units for a given class. The following, for instance, are the content objectives for the strand unit of length in 1st class (Government of Ireland, 1999a, pp. 52–53):

The child should be enabled to

- estimate, compare, measure and record length using non-standard units;
- select and use appropriate non-standard measuring units and instruments;
- estimate, measure and record length using standard unit (the metre);
- solve and complete practical tasks and problems involving length.

This listing is at variance with the idea of problem-solving providing a context within which concepts and skills can be developed. Rather, the impression given is that children first have to learn procedures and then apply these known procedures to practical situations.

In Report No. 17 we discussed mathematical proficiency as an aim of the curriculum and mathematization as a key focus. In this chapter we extend the discussion of curriculum structure by addressing aims and goals. How these elements relate to each other is an important issue in a redeveloped curriculum. The approach we take here is to give some attention to each of the key processes associated with mathematization. We also give a brief account of each of the five content domains with particular reference to key emphases in recent years.

Curriculum Aims

In their description of the landscape of learning mathematics, Fosnot and Dolk (2001) make reference to learners journeying towards a 'horizon'. We conceptualise this horizon – or aim of mathematics education – as mathematical proficiency. A redeveloped curriculum should serve to realise this aim and goals, coherent with this aim, should be identified.

Curriculum Goals

In Report No. 17 (Chapter 4, Section: [*Breaking Down the Goals: Critical Transitions within Mathematical Domains*](#)) we proposed that we first need to identify general goals and these then need to be broken down for planning, teaching and assessment purposes. A goal specification with a strong focus on processes is in keeping with a sociocultural perspective on learning. The presentation of goals in the Dutch mathematics curriculum is of interest to the Irish situation. Their starting point is the characterisation of mathematics education and a statement of core goals for the entire primary mathematics curriculum (van den Heuvel-Panhuizen & Wijers, 2005). To support progression to these goals, the Dutch team have developed learning-teaching trajectories for calculation with whole numbers (van den Heuvel-Panhuizen, 2008) and for measurement and geometry (van den Heuvel-Panhuizen & Buys, 2008). Within these trajectories are intermediate attainment targets that serve as a series of reference points against which children can be assessed. Suitable teaching methods at each stage of the learning process are

also provided in the learning-teaching trajectories. Furthermore, the targets are to be used in conjunction with the characterisation of mathematics education and core goals. The Dutch team resists an excessive precision with regard to age and grade-level. This is to avoid frequent testing of children to see if they are meeting targets. Instead trajectories are described for two successive school years in recognition of the fact that children learn at different rates.

As has been outlined in Report No. 17, the differences between the ways learning paths are presented rest largely on their theoretical underpinnings. In the US, Sarama and Clements (2009) draw heavily from the field of cognitive science, whereas the RME team draw from classroom-based research. The developmental progressions described by Sarama and Clements are finely grained and age-related, whereas the TAL⁴ trajectories are characterised by fluidity and the role of context (see [Table 5.2. A Developmental Progression for Volume Measurement](#) and [Table 5.2. A Developmental Progression for Volume Measurement](#), Report No. 17). We suggest use of learning paths to explicate critical transitions in relation to the content. We see that in such a formulation, there would be specific reference to processes throughout. In line with a sociocultural approach to the learning of mathematics, we advocate that learning paths be used in a flexible way to posit shifts in mathematical reasoning and to inform planning and assessment.

In the section below we discuss some issues pertaining to process and content-oriented goals for mathematics education. The mathematical processes discussed are those associated with mathematization (a key focus of the curriculum). The content goals discussed are those generally addressed in international curricula.

Mathematical Processes

In Report No. 17, we discussed how mathematical proficiency is developed through engagement with the processes encompassed in the overarching concept of mathematization (Bonotto, 2005; NRC, 2009). The processes – communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting – are described next.

Communicating

Communication is at the heart of mathematics learning. Zevenbergen et al. (2004) describe communication in mathematics from a multi-literacy perspective:

In terms of multiliteracies, the mathematics in a classroom is a text of which students will make interpretations (or readings). When teaching is seen in this way, it becomes possible to understand the learner as a much more active participant in the classroom and in so

4 In Dutch, learning-teaching trajectories are referred to as TALs (i.e., Tussendoelen Annex Leerlijnen).

doing allows the teacher to realise that students construct very different interpretations of what has been said or done. This moves the emphasis away from seeing students as giving right or wrong answers to one where the role of teacher becomes more of understanding why students construct responses and understandings in the ways they do. Not only are the communications related to mathematics, but so also are the texts within which the mathematics is being conveyed to the students. Meaning making becomes multi-dimensional. (p. 117)

Among the communications they identify as relevant to mathematics are:

- oral communication – contexts for this include whole-class discussion, small group work, play, dramatic performances etc.
- visual communication – this might take the form of 2-D displays, constructions, photographs
- digital communication – displays can be created using digital technology, e.g., AutoCollage and Glogster
- textual communication – this includes scribbles, drawings, stories, ways of thinking sheets etc.
- symbolic communication – this involves communicating meaningfully in the symbolic form of mathematics (e.g., +, -, =); as discussed in Report No. 17 (Chapter 2, [Sociocultural Perspectives](#)) children move from invented to conventional symbol systems.

Reasoning

While there are various accounts of mathematical reasoning (Sternberg, 1999), it is generally associated with logic and the drawing of valid conclusions (e.g., Artzt & Yaloz-Femia, 1999; Steen, 1999). Reid (2002), drawing from the NCTM *Principles and Standards for Teaching Mathematics* (2000), describes three elements that constitute mathematical reasoning in primary school settings: (a) examining patterns and noting of regularities; (b) supporting statements by showing that they apply in other cases or rejecting statements by providing counterexamples; and (c) explaining reasons 'why'. Earlier we discussed the principle of promoting a metacognitive approach as a means of helping children to monitor their own learning and development (See Chapter 1, Section: [Promoting a Metacognitive Approach](#)). Tang & Ginsburg (1999) suggest that metacognitive ability, i.e., 'thinking about one's thinking', is closely related to reasoning. As such, helping children to understand their thinking and assisting them to express it to others are central to the learning of mathematics. This expression may take many forms. For example, children might use a questioning tone to indicate uncertainty or might smile to convey their belief that they have found a satisfactory solution to a problem. Gestures such as imitating actions, intentionally using gaze, touching and pointing have been identified as key modes of expression for young children (Flewitt, 2005). Educators, therefore, need to pay close attention to issues such as tone of voice, facial expression, gesture, and specific use of words as indicators of children's self-awareness.

Argumentation

Krummheuer (1995, p. 229) describes argumentation as ‘a social phenomenon; when cooperating individuals [try] to adjust their intentions and interpretations by verbally presenting the rationale of their actions’. It is considered central to mathematics development because children have to make sense of their own explanations and the explanations of others and have to compare the claims of others against their own (e.g. Perry & Dockett, 1998, 2008; Yackel & Cobb, 1996). Perry and Dockett (1998) suggest that argumentation might occur in a variety of situations; however, play, because of its significance in the lives of young children, offers a particularly potent context in which it might emerge.

Justifying

Related to the idea of justification is that of ‘self-explanation’ described by Siegler and Lin (2010, p. 85) as ‘inferences concerning ‘how’ and ‘why’ events happen’. Drawing on a number of mathematics and science experiments with young children, they conclude that preschoolers, as well as older children and adults, can benefit from encouragement to explain their thinking. They also suggest that explaining other people’s answers can be more useful for children than explaining their own answers. Not surprisingly, the more time children are given to think about such explanations, the higher will be the quality of their learning. The authors also report that ‘I don’t know’ responses (observed in a study with 5-year-olds) decreased over time (see also Chapter 2, Section: [Promotion of Math Talk](#)). Moreover, Perry and Lewis (1999) report that verbal imprecision (e.g., false starts or long pauses) can be related to improved problem-solving performance. This imprecision and hesitation in young children’s verbal interactions have been found to often indicate their engagement with deep intellectual work (Tizard & Hughes, 1984).

Generalising

Generalisation involves a shift in thinking from specific statements to more general assertions. The fact that children use concrete objects to explore mathematical thinking does not imply that they are not engaged in abstract thought as, in the words of Russell (1999, p. 3), ‘the very nature of mathematics is abstract’. She suggests that as children learn to count, they are already dealing with abstract ideas and that this leads to further abstractions (e.g., ‘Numbers go on for ever.’). In particular, children often express generalisations using language, diagrams and story contexts (Bastable & Schifter, 2008). For example, a child might say, ‘It doesn’t matter what way you add two numbers, the answer stays the same’ to express the commutative property of addition. In this case ‘you’ is not used by the child to address another person but to convey generality – what happens ‘every time’ (Rowland, 2000).

Mason (2008) contends that children begin to generalise from an early age (for example in learning vocabulary such as 'cup', 'dog' etc.) and yet this capacity to generalise is rarely exploited in educational settings:

...as teachers, we often try to do the work for them [children]. We provide particular cases, we display methods, and we provide worked examples. We then expect them to generalize, yet rarely do we explicitly and intentionally prompt them to use their powers to generalize, nor display that power being used. (p. 64)

However, errors in children's mathematical thinking can be caused by the development of prototypes (e.g., only recognising a triangle if it is lying 'flat') or by overgeneralisation (e.g., that a smaller digit must always be subtracted from a larger one), both of which can be countered to some extent by engagement in rich and varied mathematical experiences (Ryan & Williams, 2007). Generalisation is embedded in algebraic thinking which is considered later in this chapter.

Representing

Among the forms of representation that children use to organise and convey their thinking are concrete manipulatives, mental models, symbolic notation, tables, graphs, number lines, stories, and drawings (Langrall et al., 2008). These are sometimes referred to the literature (and earlier in this volume) as 'tools' (see, for example, Anthony & Walshaw, 2007); in other words, the terms are often used interchangeably.

Meira (2002) argues that representations are at the heart of sense-making in mathematics:

It is often the case... that mathematics instruction not only restricts students' production of unconventional and specialized notational systems (e.g., when doing arithmetic on paper), but also aims at suppressing the students' 'dependency' on representations altogether (often viewed only as a means to acquire mental competencies). Tallies and diagrams on paper (as well as finger counting and the use of hand calculators) are not lesser means of doing mathematics, but the very material basis of sense-making. (p. 102)

While representations in their many forms are integral to children's mathematical sense-making, there are some caveats that must be taken into consideration in their use. For example, it has traditionally been considered that there is a linear development from concrete to abstract thinking (Piaget, 1952). The literature on the subject suggests that this is not necessarily the case and that representations or models can sometimes inhibit children's mathematical thinking (for example, Uttal, Scudder, & DeLoache, 1997). This is the case because children do not necessarily understand the relationship between the model (e.g., Dienes' blocks) and the mathematical concept that they are supposed to represent (e.g., place-value). Uttal et al. (1997) suggest that models or concrete manipulatives can be seen in two ways: as objects in their own rights and as representations of

something else. The more children treat the materials as objects, the less likely it is that they will discern underlying mathematical concepts. Boulton-Lewis (1999) make a similar point. She suggests that what is really needed is for children to be very familiar with the objects so that the focus of activity is on deepening mathematical understanding rather than on the features of the materials. In other words, children should have ample opportunity to explore through free play the full extent of the materials prior to mathematical discussions. According to Perry and Dockett (2008), the RME interpretation of modeling where models *of* become models *for* mathematical reasoning is preferable (see Chapter 2, Section: [Emphasis on Mathematical Modeling](#)).

Problem-Solving

Although problem-solving is accorded a central role in the PSMC, it continues to be an area in which children in Ireland underperform (see Report No. 17, Introduction, Section: [Performance Context](#)). As argued above, much of this rests on the fact that problem-solving is often used as a means of practising acquired skills rather than a context in which to learn mathematics. In relation to this, Hiebert et al. (1996, p. 12) talk about the need to make the subject problematic:

Allowing the subject to be problematic means allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas and questions for students. We do not mean 'problematic' to mean that students should become frustrated and find the subject overly difficult. Rather we use 'problematic' in the sense that students should be allowed and encouraged to problematize what they study, to define problems that elicit their curiosities and sense-making skills. (p. 12)

They suggest that three kinds of understanding remain ('residue') after a problem is solved: insights into the structure of mathematics, strategies for solving problems, and dispositions towards mathematics. In other words, through engaging in problem-solving, children not only learn problem-solving strategies but also deepen their understanding of mathematics. While play, modeling activities, project work as well as open-ended tasks and other practices discussed in Chapter 2 can be used as contexts for problem-solving, all topics should be introduced to children as 'problematic'. For example, the addition or subtraction of two-digit numbers can be explored via a problem where children are encouraged to construct non-standard algorithms that reflect their developed understanding of place-value (e.g., Cobb, Yackel, & Wood, 1992; Fosnot & Dolk, 2001).

Connecting

The notion of 'connections' in mathematics relates both to those that exist: (i) within and between different content areas in mathematics (e.g., within number or between number and measurement); (ii) between mathematics learning and learning in other areas; and (iii) between mathematics and the context within which a child lives, works or plays (Perry & Dockett, 2008). The idea of

connections within mathematics receives considerable treatment in the US NRC (2009) report where it is stated that 'every mathematical idea is embedded in a long chain of related ideas' (p. 48). Johanning (2010) proposes that, in order to build a coherent curriculum and to foster connections, the big ideas from one topic must be built on in others so that children are given the opportunity to use familiar concepts in new settings.

We can see that these processes are strongly interconnected and that they are integral to the development of a mathematics-learning community. They should characterise and be promoted through math talk, that is, children engaging in reasoning, argumentation, justification etc. While we have suggested both in Report No. 17 and earlier in this chapter that a rebalancing of the focus on processes compared with content is required in a revised curriculum, that is not to suggest that content is unimportant. Below we give an overview of the content areas that are found in mathematics curricula for young children internationally, although the labels and levels of emphasis may vary.

Content Areas

In the PSMC, six strands are specified for children in infant classes – Early Mathematical Activities, Number, Algebra, Shape and Space, Measures and Data. For children at higher class levels, the last five of these comprise the content areas. We advise that these five continue to be the broad areas of content in the revised curriculum. However, they should be explicated in ways that reflect current research, and developments in curriculum structure and design. The strand units of Early Mathematical Activities, i.e., Classifying, Matching, Comparing and Ordering, are now generally addressed within each of the other content areas (e.g., NRC, 2009; Sarama & Clements, 2009; van den Heuvel-Panhuizen, 2008).

In the NRC (2001) report, much attention is given to the domain of number which the authors contend lies at the heart of other strands. However, they emphasise the need to develop mathematical proficiency across all strands of the curriculum:

Students need to learn to make and interpret measurements and to engage in geometric reasoning. They also need to gather, describe, analyze, and interpret data and to use elementary concepts from probability. Instruction that emphasizes more than a single strand of proficiency has been shown to enhance students' learning about space and measure and shows considerable promise for helping students learn about data and chance.
(p. 8)

There follows a brief account of each of the five content domains referred to above. While an in-depth treatment of each content area is beyond the scope of this report, some important emphases that need to be taken into consideration in a redevelopment of the mathematics curriculum are identified.

Number

As mentioned in Report No. 17, emphasis needs to be placed on the development of 'number sense', described by Anghileri (2000) as follows:

It is not only effort that gives some children a facility with numbers, but an awareness of the relationships that enable them to interpret new problems in terms of results they remember. Children who have this awareness and the ability to work flexibly to solve number problems are said to have a 'feel' for numbers or 'number sense'. What characterises children with 'number sense' is their ability to make generalizations about the patterns and processes they have met and to link new information to their existing knowledge. (p. 1)

Sarama and Clements (2009) identify the components of number sense as composing and decomposing numbers, recognising the relative magnitude of numbers, using benchmarks, linking representations, understanding the effects of operations, inventing strategies, estimating, and possessing a disposition toward making sense of numbers. Among the 'big' ideas about number that are considered important for the 3- to 8-year-old children are counting, comparing, unitising, grouping, partitioning, and composing.

Arising from a review of the literature, Dunphy (2007) presented the following framework reflecting key aspects of number sense as it relates to 4-year-old children:

- pleasure and interest in numbers (disposition)
- understandings of some of the purposes of numbers (as derived from everyday experiences)
- quantitative thinking (e.g., counting, relating numbers to other numbers, subitising, estimating), and
- awareness and understanding of written numerals (based on interactions about numerals).

There are different approaches to specifying details of the content area of number. Van den Heuvel-Panhuizen (2008, p. 11) concentrates on calculations with whole numbers. The crucial developmental steps that children aged between 2 and 8 take are identified by means of reference points (Intermediate Attainment Targets). She argues that this approach offers the possibility for teachers of '...a helicopter view, the possibility of grasping, in a few large-scale steps, the course of development that takes place'.

The NRC (2009) report identifies three interrelated aspects of early number including whole number, relations, and operations. In relation to each of these, a sequence of milestones for children aged 2–7 are identified. These correspond to the more detailed specification offered by Sarama and Clements (2009) who provide a comprehensive overview of the various elements of content in the number strand. These comprise of:

- quantity, number and subitising
- verbal and object counting
- comparing, ordering and estimating
- arithmetic: early addition and subtraction and counting strategies
- arithmetic: composition of number, place value, and multi-digit addition and subtraction.

For each of these, the authors provide developmental progressions, linked to age. While we see the linking of critical concepts with ages as greatly problematic (see Report No. 17, Chapter 5, Section: [Recognising Developmental Variation](#)), nonetheless the developmental progressions suggest important concepts that children need to develop.

Some of the key emphases for counting, as deduced from *A Developmental Progression for Counting* (Sarama & Clements, 2009, pp. 73–79), but also informed by van den Heuvel-Panhuizen (2008) and the NRC (2009) report are as follows:

- verbal counting
- making 1–1 correspondence between items and number words (touch counting)
- meaningful (object) counting of small groups (linear)
- answering the question ‘How many?’
- meaningful counting of small groups (random arrangement)
- recognising the purposes for which counting is useful
- using fingers to symbolise
- learning the order of counting words beyond 10, beyond 20, to 100 and beyond as required
- counting from a specific number
- counting backwards
- skip counting
- counting imaginary objects.

Children progressively extend the range of their counting. In doing so, they demonstrate increasing interest, focus and effort. As children learn to count (and this learning continues right across the age span 3–8 years), they draw on the other main quantification strategy of subitising. They also make connections to other developing concepts, processes and skills including those related to cardinality and ordinality.

The educator's task is to guide the learning process. The ideas about number indicated above are important. Important too are operations on numbers such as addition and subtraction and procedures for carrying out the operations (NRC, 2005). While international curricula offer different specifications as regards number caps, we conclude that the key issues relate to emphasis on number analysis and key transitions in number analysis (e.g., grouping in tens, addressing 'teens' etc.), rather than number caps per se. Such a position is in keeping with a sociocultural perspective. The various progressions as explicated in the literature offer curriculum designers research-based frameworks which can be drawn on and from which they can extract the important concepts within each aspect of number (and other strands).

Measurement

Measurement is an important mathematical topic because of its applicability to everyday activity, because of its connections with other subject areas and because it can serve as the basis of other content areas in mathematics (Clements, 2003). However, the difficulties inherent in learning measurement concepts should not be underestimated – in particular, measurement differs from number concepts in that it involves the subdivision of continuous quantities into units (Outhred, Mitchelmore, McPhail, & Gould, 2003). A broad outline of appropriate early measurement experiences is as follows (Sarama & Clements, 2009, pp. 274–275):

- encountering, discussing and using appropriate vocabulary for quantity or magnitude of a certain attribute
- comparing two objects directly and recognising equality or inequality
- overcoming perceptual cues and developing the capacity to reason about and measure quantities.

However, each of the topics within measurement presents particular cognitive challenges that need to be addressed. For example, among the challenges that young children encounter in linear measurement are the need to use equal size units, the fact that differing size units lead to different numerical answers (whilst the actual measure is preserved) and the inverse relationship between size of unit and the number of units required for the measure (NRC, 2009). The central concepts in linear measurement are shown in Table 3.1:

Table 3.1: Central Concepts in Linear Measurement

	Idea	Description
Conceptions of unit	Iteration	A subdivision of a length is translated to obtain a measure.
	Identical unit	Each subdivision is identical.
	Tiling	Units fill the space.
	Partition	Units can be partitioned.
	Additivity	Measures are additive, so that a measure of 10 units can be thought of as a composition of 8 and 2, and so on.
Conceptions of scale	Zero-point	Any point can serve as the origin or zero-point on the scale.
	Precision	The choice of units in relation to the object determines the relative precision of the measure. All measurement is inherently approximate.

Source: Lehrer, Jaslow, & Curtis, 2003, p. 102

Sarama and Clements (2009) draw the following conclusions about geometric measurement:

- While it is generally assumed that children learn length first, then area and then volume, this sequencing only applies to the 'spatial structuring' aspects of these measures, e.g., in order to cover a two-dimensional space (area) with units, the child needs to understand the covering of a one-dimensional space (length). In particular, even older primary school children can find 'packing' volume quite challenging. However, other aspects of these measures can develop in parallel (e.g., using a container to measure liquid volume).
- Although there is evidence that young children can develop ideas about attributes such as angle and area from an early age, there is little research to support the investment of time in these topics rather than others.
- While children first develop ideas about measuring different attributes, it takes both time and high-quality educational experiences for them to generalise ideas about measurement across attributes.

There is also evidence that young children can develop concepts of non-geometric measurement such as weight (e.g., Cheeseman, McDonough, & Ferguson, 2012) and time (e.g., Kamii & Long, 2003), although the need to make connections to children's everyday lives and to present stimulating contexts for the learning of each of these topics is stressed. Across all measurement topics, there is some debate about the merits of starting with non-standard units and not with

standard measuring devices (e.g., Cheeseman, et al., 2012; Stephan & Clements, 2003). In the NRC (2009) report, research is cited showing that children are often more successful at measuring (length) using standard rather than non-standard units and devices, that using non-standard units actually detracts from children's understanding of basic measurement concepts, that use of a conventional ruler can support mathematical reasoning about length more effectively than non-standard instruments, and that children often show a preference for standard devices. Such findings point to the need to re-examine the linear progression from non-standard to standard units and devices of measurement in the current PSMC.

Geometry and Spatial Thinking

According to Sarama and Clements (2009), geometry and spatial thinking is the second most important area in mathematics learning for young children after number, not only because geometric concepts are important in their own right but also because they support number and arithmetic concepts and skills. Sarama and Clements consider geometric content from three perspectives: (i) the space in which the child lives, (ii) geometric shapes (2-D and 3-D) and (iii) composition and decomposition of shapes. In their consideration of shapes they refer in particular to the work of Pierre and Dina van Hiele who posited five levels of geometric thinking⁵, two of which are relevant to young children:

- Level 0 (Visualisation). The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual considerations of the concept as a whole without explicit regard to properties of its components.
- Level 1 (Analysis). The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established (Burger & Shaughnessy, 1986, p. 31).

The levels were considered by the van Hieles to be sequential, discrete and hierarchical (van Hiele, 1959/1984). However, this static view of the levels has been disputed (e.g., Burger & Shaughnessy, 1986). Gutiérrez, Jaime and Fortuny (1991) propose that students use different levels of reasoning depending on the problem to be solved and that there are degrees of acquisition within each level. Clements, Swaminathan, Hannibal and Sarama (1999) maintain that a pre-cognitive level exists before the visual level where children cannot distinguish (2-D) shapes such as circles, rectangles and triangles from non-exemplars of classes of these shapes. According to Clements et al., these children 'are in *transition to*, instead of *at*, the visual level' (p. 205, italics in original). They also

5 The van Hiele model of geometric thinking was developed by Dina van Hiele-Geldof and Pierre van Hiele who completed their doctoral theses on the subject at the University of Utrecht, The Netherlands in 1957. The English translations of the major works of the couple brought the model to the attention of US scholars in the 1970s (Crowley, 1987).

advocate a renaming of the visualisation level as 'syncretic', since a level does not consist of 'pure' forms of knowledge – for example, visualisation includes both visual/imagistic knowledge and declarative knowledge ('knowing what').

Sarama and Clements (2009) provide detailed learning paths for children from birth to 8 years in the following aspects of shape and space:

- spatial thinking (with separate paths for spatial orientation and spatial visualisation and imagery)
- shape
- composition of 2-D shapes
- composition of 3-D shapes
- embedded geometric figures.

A feature of their learning paths is the use of descriptive terms to describe the processes of children at different levels. Hence, in the case of spatial visualisation, we find simple sliders (who can move shapes to a location), simple turners (who can mentally turn objects in easy tasks), beginning sliders, flippers and turners (who can use correct motions, but not always accurately), more advanced sliders, flippers and turners (who can perform slides and flips, using manipulates, and make turns of 45, 90 and 180 degrees), diagonal movers (who can perform diagonal slides and flips), and mental movers (who can predict results of moving shapes using mental images).

Another description of the progression of 3- to 5-year-olds in geometry/spatial thinking can be found in the NRC (2009) report, which provides learning paths for space and shape in two dimensions, and in three, with each learning path focusing on describing and constructing objects, spatial relations, and compositions and decompositions. The NRC report highlights the importance of providing young children with substantial experience of shape and space, and warns that, if the shape categories that children experience are limited, so will their concepts of shapes. One implication of this is that children need to encounter 'rich and varied examples and non-examples, and discussions about shapes and their characteristics' (p. 192). The NRC (2009) report also suggests a range of activities designed to support children's development of spatial thinking.

The learning paths provided by Sarama and Clements and by the NRC are quite detailed, possibly because the authors feel that teachers in the United States may themselves have limited understanding of geometry and spatial reasoning, and hence might benefit from a high level of detail. An alternative approach is evident in the work of van den Huevel-Panhuizen and Buys (2008) who provide intermediate learning targets in geometry for children in kindergarten 1 and 2 (junior and senior infants) and grades 1 and 2 (first and second classes). The intermediate learning targets are brief narrative descriptions of mathematical processes children at each grade range can be expected to engage in. There are three targets at each grade range – one each covering orienting (describing position in space), constructing, and operating with shapes and figures. The intermediate

learning targets are accompanied by descriptions of the associated mathematical reasoning, and of activities that might be presented to children to support their development. Many of the activities are rooted in children's everyday experiences, or are embedded in fictional stories that provide realistic contexts for activities such as map-making.

van den Huevel-Panhuizen and Buys (2008) also note a link between geometry/spatial reasoning and a range of goals of primary education including:

- developing a positive working attitude
- making connections between mathematics and daily life
- making practical applications
- reflecting on one's own mathematical activities
- developing and designing connections, rules, patterns and structures.

They focus, in particular, on the aesthetic value of geometry (making patterns, use of symmetries, discovering structure in nature, developing an eye for geometric elements in art, design and architecture), which, they argue, can contribute to the cultural development of primary school children, as well as developing their mathematical proficiency.

Algebraic Thinking

While there are many perspectives on the nature of algebra and particularly what might constitute algebraic thinking in the early grades (e.g., Cai & Knuth, 2011; Kaput, Carraher, & Blanton, 2008), Kieran (2011, p. 581) suggests the following themes as those that predominate recent research literature on the subject:

- Thinking about the general in the particular – this idea stems in particular from the work of John Mason (mentioned in the section on '[Generalising](#)' above).
- Thinking rule-wise about patterns – this concerns not just determining a commonality in a sequence but extending the rule to indeterminate quantities.
- Thinking relationally about quantity, number and number operations – this involves seeing numbers and number operations in terms of their inherent structural relations (e.g., $8 + 5 = 10 + 3$).
- Thinking representationally about the relations in problem situations – this pertains to using a variety of representations (e.g., context, manipulatives, drawings) to visualise a problem situation.
- Thinking conceptually about the procedural – this approach to mathematical procedures implies a focus on rich mathematical connections, generalities and relationships that emanate from the procedure.

- Anticipating, conjecturing, and justifying – in particular this concerns the development of a classroom culture where questions are used by the teacher to move students forward in their thinking; where students explain and justify their reasoning and where they delve into challenging mathematical ideas.
- Gesturing, visualising, and languaging – young children draw on a variety of ways – visual, aural, motor senses – to express pattern regularity (e.g., they might use gesture and /or words to signify a ‘non-present’ element of the pattern).

What is apparent from this list is that many of the characteristics of algebraic thinking are analogous to the processes (e.g., communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting) described earlier. From this perspective, algebraic thinking serves to give a deeper treatment to other content domains. Bastable and Schifter (2008) put it like this:

When the arithmetic classroom environment is designed to follow children’s thinking and provides elementary students with the opportunity to pursue their own questions, they display interest and ability in formulating and testing generalizations. Although these students do not, of course, use conventional algebraic symbols to express their ideas, the kinds of arguments they pose and the kinds of reasoning they display have parallels in formal algebra. (p. 165)

Thus the infusion of algebraic thinking across the mathematics curriculum would facilitate the development of the processes.

In terms of pattern work, children should be given the opportunity to explore a wide range of materials. Their attention can be drawn to the many patterns in nature and in their everyday environment. Initially in preschool, children should explore sequences since the ability to recognise sequences is important in pattern work. When children recognise that repeating sequences form a pattern, they can begin to organise their pattern making. This can focus on different attributes, e.g., size, colour, shape, orientation etc. They can deal with both pattern making and pattern perception but appear, initially, to find it easier to talk about the characteristics of patterns that they have created themselves than to discuss those created by others (Garrick, Threlfall, & Orton, 1999). As they grow older, there needs to be a focus not only on creating and recognising patterns but also on increasing the complexity of patterns (Threlfall, 1999). Later children can move towards describing a pattern numerically.

In order to encapsulate the breadth of the domain of algebraic thinking, Cooper and Warren (2011) suggest a framework for curriculum that encompasses (i) pattern and functions, (ii) equivalence and equations and (iii) arithmetic generalisation. In the revised curriculum, consideration could be given to explicating these in two strands, i.e., Algebra and Pattern, and Number (e.g., ACARA, 2009).

Data and Chance

Data is the domain that receives least attention in research on mathematics education in the early years (Clarke, 2001; Sarama & Clements, 2009). Sarama and Clements (2009) suggest that in order for children to understand data analysis they must learn concepts of ‘expectation’ (e.g., averages, probability) and ‘variation’ (uncertainty, spread of values). Jones et al. (2000) formulated a framework for characterising children’s statistical thinking. The four constructs in the framework are ‘describing’, ‘organising’, ‘representing’ and ‘analysing and interpreting’ data. For each construct there are four thinking levels – idiosyncratic, transitional, quantitative and analytical – on a continuum. In a small scale study of 20 children from grades 1 – 5 (US), they found that children in grades 1 and 2 typically exhibited thinking at level 1 (idiosyncratic) or level 2 (transitional)⁶. They also found lowest levels of thinking on the ‘analysing and interpreting’ construct, a finding they attribute to possible poor focus on this construct in classroom activities. Leavy (2008) suggests that children’s ownership of a statistical problem is a critical factor in developing their statistical reasoning beyond level 1. In this regard, it is interesting that in the recently developed Australian mathematics curriculum (ACARA, 2009), the content domain is termed ‘statistics and probability’ rather than ‘data and chance’ in order to emphasise the need for children to interpret and analyse as well as represent and summarise data. While probabilistic reasoning (‘chance’) has not traditionally featured in mathematics curricula for children aged 3–8 years because of the cognitive challenges that it poses, it now tends to be included from kindergarten on. The emphasis is on language development, e.g., ‘might’, ‘maybe’ and the need to ground understanding in children’s everyday lives (e.g., Metz, 1998).

Content Areas and Curriculum Presentation

Linkages

Although there is generally broad agreement on the content domains listed above, in recent curricula there has been a tendency to amalgamate some of the domains. For example, in the study of measurement and geometry there are some considerable overlaps. Such overlaps also exist between measurement and data, number and algebra etc. The domains listed for US Common Core States Standards for Mathematics (CCSM) are as follows (White & Dauksas, 2012):

- Operations and Algebraic thinking
- Number and Operations in Base ten
- Measurement and Data
- Geometry.

6 In representing data, for example, a child at thinking level 1 would produce an invalid or idiosyncratic display of a data set while a child at thinking level 2 would produce a display that is partially valid.

The content strands in the Australian curriculum (ACARA, 2009) are:

- Number and Algebra
- Measurement and Geometry
- Statistics and Probability
- Geometry and Measure.

As mentioned earlier, consideration also needs to be given to the idea of an integrated curriculum in which mathematical concepts and skills are developed across a class-level rather than repeating ideas from year to year (Johanning, 2010). For example, certain content areas might be emphasised at a particular class level but over an extended period, e.g., two years, all content areas would receive attention. This allows for more in-depth exploration of key topics.

Learning Outcomes

In the revised Dutch curriculum, the goals describe opportunities to learn rather than intended competencies or content objectives. This subtle shift has important consequences. If mathematical content is framed as a list of competencies, the result is narrow and basic since the content has to apply to all students. Opportunities to learn, on the other hand, give more scope to describe what is considered important for students to learn – ‘whether they actually will learn this...cannot be fixed’ (van den Heuvel-Panhuizen & Wijers, 2005, p.293). This places more focus on learning than on attainment. In this regard, the call in the *National Strategy to Improve Literacy and Numeracy* (DES, 2011a) for the use of learning outcomes as opposed to content objectives is welcome. As described in Report No. 17 (Chapter 4, [Table 4.1. Specifying Goals: Different Approaches](#)), in the *US Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006), critical ideas are broken down into transitions that indicate shifts in mathematical reasoning. These narrative descriptors, together with goals and learning paths, contribute to the formulation of learning outcomes. Such an approach might provide a basis for structuring the curriculum at content level, with the content-level descriptors providing a basis for identifying learning outcomes. Figure 3.1 shows an emerging curriculum model highlighting how the relationships between the different elements may be conceptualised.

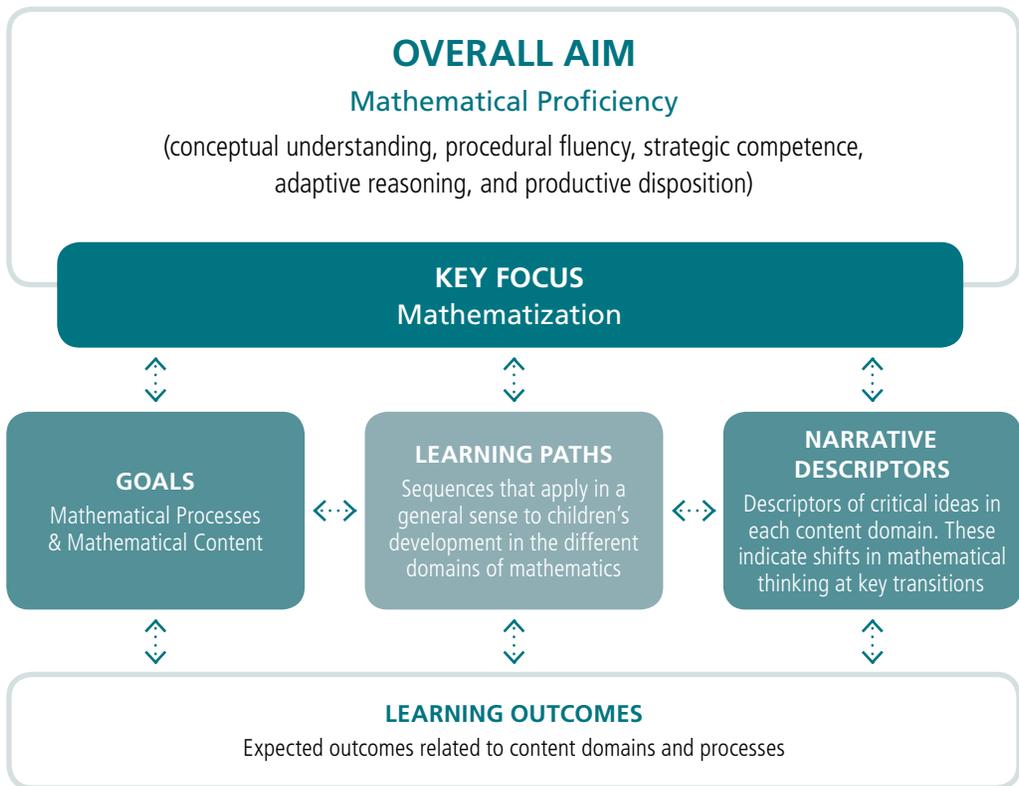


Figure 3.1: Emerging Curriculum Model

Conclusion

In this chapter we have given consideration to ways in which the mathematics curriculum for 3- to 8-year-old children might be developed. In particular, we have argued for the development of a coherent curriculum where there is close alignment between the aim of mathematical proficiency and goals related to processes and content. Engagement with processes of communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting serves to deepen children's mathematical learning. The content areas related to Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance – in whatever way they are labelled – constitute the mathematical knowledge with which children should engage.

The key messages arising from this chapter are as follows:

- Goals of the curriculum should relate both to processes and content.
- The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded.
- In line with the principle of 'mathematics for all', each of the five domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance – should be given appropriate attention.
- While critical ideas in each content domain need to be explicated, over-specification or age-specification should be avoided.
- Narrative descriptors of critical ideas indicating shifts in children's mathematical reasoning are potentially useful for teachers.
- Learning outcomes, derived from narrative descriptors, are a preferred alternative to content objectives.

Research Report No. 18

Mathematics in Early Childhood and Primary Education (3–8 years)

CHAPTER 4

Curricular Issues



In this chapter we consider a number of curricular issues related to curriculum implementation and effective mathematics pedagogy. These include provision of an equitable mathematics curriculum that is inclusive of all children; early intervention; allocation of time to teaching mathematics; and integration of mathematics across the curriculum. In considering equity issues, we discuss the needs of exceptional children, including those with intellectual and developmental difficulties, and children with mathematical talent. We also focus on children in culturally-diverse contexts including English language learners, children learning mathematics through Irish, and children living in disadvantaged circumstances.

An Equitable Curriculum

The vision of equity in a curriculum ‘challenges teachers to raise expectations for the mathematical learning of all students and to provide instruction that responds to students’ prior knowledge, academic strengths, and individual interests’ (Lloyd & Pitts Bannister, 2010, p. 324). In Report No. 17, we discussed how ‘mathematics for all’ implies a pedagogy that is culturally sensitive and takes account of individuals’ ways of interpreting and making sense of mathematics. Furthermore, we highlighted that exceptional children (those with developmental disabilities or who are especially talented at mathematics) do not require distinctive teaching approaches but should have their individual needs met (see Report No. 17, Chapter 7, Section: [Exceptional Children](#)).

The view of mathematics that is integral to this report and to Report No. 17 means that the mathematics curriculum is not neutral and objective but is mediated by culture. An implication of this is that the mathematics curriculum and pedagogy have to take account of learners’ interests, backgrounds and ways of knowing. Among the ways that Ladson-Billings (1995) identifies as useful in fostering a multicultural curriculum and pedagogy that embraces the needs of all students are the following:

- The importance of treating all children as if they *already* have knowledge and experience that can be used as a foundation for teaching.
- The creation of a learning environment that allows children to move from what they do not know to what they do know.
- A focus on high-quality mathematics learning rather than on ‘busy’ work.
- The provision of challenging tasks to *all* children.

- The development of in-depth knowledge of children and subject matter.
- The fostering of strong teacher-child relationships.

These pedagogical features are also consistent with those identified in Chapter 2. In this regard, the strategies used to cater for the diverse needs of learners constitute 'good teaching', helping children to 'see mathematics as a human endeavour done by real people to serve real needs and interests' (Wiest, 2001, p. 22). A key issue in developing an inclusive classroom is the philosophical orientation of the teacher. This includes adherence to the belief that 'it is helpful to view difficulties in learning as problems for teachers to solve rather than problems within learners' (Florian & Linklater, 2010, p. 371). In addition, it demands that in schools there is an 'ethic of everybody: teachers have both the opportunity and responsibility to work to enhance the learning of all' (Florian & Linklater, 2010, p. 372).

Exceptional Children

As noted in Report No. 17 (see Chapter 7, Section: [Exceptional Children](#)), Kirk, Gallagher, Coleman and Anastasiow (2012) define as 'exceptional' a child who differs from the 'typical' child in (i) mental characteristics, (ii) sensory abilities, (iii) communication abilities, (iv) behaviour and emotional development, and/or (v) physical characteristics. The term includes both children with developmental delays and those with gifts and talents.

Children with Intellectual and Developmental Difficulties

In the literature on inclusive strategies, the question arises concerning the extent to which specialised approaches are needed for some children with special educational needs. In a review of pedagogies for inclusion, Lewis and Norwich (2005) proposed the notion of continua of common teaching approaches that can be subject to various degrees of intensity depending on individual need. However, they also state that 'in advocating a position that assumes continua of common pedagogic strategies based on unique individual differences, we are not ignoring the possibility that teaching geared to pupils with learning difficulties might be inappropriate for average or high attaining pupils' (p. 6). An example of intensification of a common teaching approach is that used by Staves (2001) in teaching counting to children with a moderate general learning disability. He suggests, among other strategies, those of applying different facets of attention such as using a small torch to point to objects in turn; increasing the emphasis of motion, rhythm or pressure when reaching the last object when guiding a child's finger; and varying the intensity of volume and vocal tones for the last number in a sequence (Staves, 2001).

Children with Hearing Impairment

Recent research has demonstrated that 'deaf children have different knowledge, learning styles and problem-solving strategies than hearing children. Teachers need to know how their deaf students

think and learn if they are to accommodate their needs and utilise their strengths' (Marschark & Spencer, 2009, p. 210). Recommendations for deaf and hard-of-hearing children include recognising their visual-spatial orientation, which they do not always apply, and their relative lack of confidence in problem-solving. Children with hearing loss 'face special difficulties when needing to relate multiple bits of information and to identify relationships' (Marschark & Spencer, 2009, p. 140). They suggest that 'it is clear that modifications in curricula and in teaching strategies are required if deaf and hard-of-hearing students are to develop to their potential in the important areas of maths and concepts' (p. 140). Interventions which have shown promise include those which focus on building problem-solving skills through producing schematic illustrations emphasising visual-spatial over verbal activities (Nunes, 2004).

Children with Visual Impairment

Access to a mathematics curriculum for children with visual impairment often hinges on specialist teacher knowledge of the unique aspects of mathematics education for such children. This includes use of calculation with abacus or braillewriter, talking calculator, concrete materials and tactile displays and teaching of the Nemeth Code (Kapperman, Heinze, & Sticken, 2000). A focus on mathematical language and its accuracy by the educator is also stressed.

Children with Autistic Spectrum Disorders

- Sensitivity to the individual needs in mathematics of children with autistic spectrum disorders (ASD) is also essential for teachers. Such children may not join in class counting activities and may find *counting on* difficult. In guidance to schools, the Department for Education and Skills (2001, p. 1) in England suggests that children who find imaginative play and play with others difficult may not have built up a wide store of mathematical concepts through engagement in such activities. Therefore a wide range of structured contexts must be provided to support the development of concepts and language. Children with ASD find some illustrations confusing and teachers may need to explain these using appropriate language.

Mathematically-talented Children

Mathematically-talented children are those who have very high levels of competence in mathematics, and can solve mathematical problems that could be considered advanced for their class level. One way in which the needs of these children might be met is through the use of 'tiered assignments' (Fiore, 2012) – that is, parallel tasks that have different levels of depth, complexity and abstractness, and different support elements or guidance, though all children work towards the same general learning outcomes. For mathematically-talented children, tasks may be differentiated by including more complex numbers, by adding obstacles to the solution process, by requiring children to engage in novel solution strategies, or by requiring them to use particular representations.

A related approach, 'curriculum compacting' (Reis, Burns, & Renzulli, 1992; Renzulli, 1994) involves (i) defining the goals and outcomes of a particular unit or segment of instruction; (ii) determining and documenting which students have already learned most or all of a specific set of learning outcomes; and (iii) providing extension strategies for material already learned through the use of instructional options that enable a more challenging and productive use of the child's time. There are other strategies for presenting the curriculum to mathematically-talented children including:

- Introducing mathematical ideas beyond those typically addressed for their age group or class.
- Developing the self-assessment and self-regulation skills needed for planning, self-assessment, monitoring, and evaluating learning activities.
- Providing a wider range of open-ended investigatory tasks.
- Providing tasks that are of interest such as those involving very large numbers, abstract mathematical explorations, and applications of mathematical ideas in a broader range of contexts.

In this view, a child's prior knowledge and strengths should guide the selection and implementation of tasks, rather than a mainly age- or grade-level focused curriculum. All of these approaches recognise that mathematically-talented children should be supported in deepening their understanding of the existing curriculum rather than being provided with an alternative one.

Children in Culturally Diverse Contexts

In this section, we consider the curricular needs of children in culturally diverse contexts, including English language learners, children learning mathematics through Irish, and children in socio-economically disadvantaged contexts.

English Language Learners

Much of the research on developing children's mathematical discourse has been conducted in settings involving young English language learners: children whose first language differs from the language of instruction. For example, Hufferd-Ackles et al. (2004) demonstrated how teachers of English language learners in third grade successfully made the transition from traditional, teacher-led pedagogy to a math talk community over the course of a school year, albeit with a modified curriculum and weekly support from a university-based mentor. Prediger, Clarkson and Bose (2012) identify three broad pedagogical strategies that are relevant to teaching mathematics to language learners. First, they highlight code switching as an important resource, in that it provides a comfortable and flexible mode of communication, and enables simultaneous learning of language and mathematics. Second, they emphasise a need to support young children in making the transition from the everyday language to a technical or mathematical register, which, they argue, can enhance children's understanding of mathematical concepts and ideas. In doing so, they note

that children may have registers for everyday language, school language, and mathematical/technical language in both their home language and in the language of instruction. Third, they stress a need to facilitate transitions between different mathematical representations – for example, between pictorial and symbolic representations, or verbal and written representations – in order to build conceptual understanding. For example, they argue that a pictorial representation can ease the language burden during initial presentation of a topic or problem, and that the emphasis can proceed to language after the underlying concept has been learned.

The literature acknowledges that second-language learners can encounter particular challenges in math talk learning communities (Chapin et al., 2009). Without an understanding of the relevant vocabulary, syntax and grammar, such learners may be prevented from demonstrating the depth of their understanding and from engaging productively in various learning activities such as explaining solution processes, describing conjectures, proving conclusions and presenting arguments, in both verbal and written contexts. Moschkovich (1999) emphasises that, when the goal is supporting children’s engagement in mathematical discussion, listening and responding to the quality of mathematical discourse is as important as focusing on children’s language proficiency, and aspects of language that relate to mathematics can be attended to from within a content-focused discussion. According to Moschkovich, the instructional strategies that might be used to attend to language in mathematics content contexts include:

- using several expressions for the same concept
- using gestures and objects to clarify meaning
- accepting and building on children’s responses
- re-voicing children’s statements using more technical (mathematical) terms
- focusing on mathematical content and argumentation.

Children Learning Mathematics through Irish

The outcomes of the *2010 National Assessments of English Reading and Mathematics in Irish-Medium Schools* point to challenges that teachers in Gaelscoileanna encounter in providing instruction in mathematics through the medium of Irish (see Gilleece, Shiel, Clerkin, & Millar, 2012). Whereas performance in mathematics was ahead of the national average at second class, it did not differ significantly from the national average at sixth class, with higher-achieving children failing to maintain the advantage enjoyed by their counterparts in second class. At sixth class, performance was above national standards on procedural aspects of mathematics, but not on reasoning and problem-solving. Teachers of 20% of children in second class in Gaelscoilenna, and teachers of 80% of children in sixth class reported teaching mathematics in both English and Irish. Teachers attributed this shift in emphasis to a need to prepare children for learning mathematics in English-medium post-primary schools and to support children who might struggle to acquire important mathematical concepts through Gaeilge.

The *2010 National Assessment of Mathematics* did not gather information on the extent to which children were engaged in math-talk learning communities, such as those envisaged in the current report. However, a study by Ryan (2011) indicates that some children in Gaelscoileanna experience difficulty in using mathematical terminology and in articulating higher-order concepts and reasoning processes as they engage in problem-solving in small group contexts. The outcomes of the national assessment and of Ryan's study suggest a need for Gaelscoileanna to pay particular attention to promoting opportunities for children to develop and use mathematical language as they engage in reasoning and problem-solving processes, in mathematics classes, and, more generally, across the curriculum.

While the development of mathematical language in immersion settings such as those found in Irish-medium schools is undoubtedly complex and may relate to a range of policy, teacher and child factors, the literature suggests that opportunities to generate, discuss, explain and justify mathematical ideas benefit all children in their development of mathematical language, regardless of the language in which instruction is provided (Barwell, Barton, & Setati, 2007; Gutiérrez, Sengupta-Irving, & Dieckmann, 2010; Ryan, 2011).

Another group of children who may learn mathematics through Irish are those attending schools in Gaeltacht areas. Gilleece et al. (2012) showed that, while children in second class in Gaeltacht schools achieved a mean score that was not significantly different from the national average, children in sixth class achieved a significantly higher average score. Importantly, Gilleece et al. reported that 45% of children in second class in Gaeltacht schools were taught mathematics in Irish only, while the remainder were taught through a combination of English and Irish. By sixth class, 50% were taught through Irish, and 50% through a combination of English and Irish. These data point to variation in competence in Irish among children in Gaeltacht schools. They also reflect the efforts of schools and teachers to adjust their use of language to take that range into account. In terms of curriculum development, it is important to support children in Gaeltacht schools as much as possible in accessing the full mathematics curriculum in the Irish language. We suggest that a strong focus on mathematical discourse from the beginning of children's schooling can play a significant role in achieving this (see Report No. 17, Chapter 3, Section: [The Role of Language in Developing Mathematical Knowledge](#); this report, Chapter 2, Section: [Promotion of Math Talk](#)).

Children in Socioeconomically Disadvantaged Contexts

As noted in Report No. 17 (Chapter 3, [Variation in Language Skills and Impact on Mathematics](#)), less advantaged children, prior to attending school, typically use the same informal strategies to solve addition and subtraction problems, they perform at about the same level as more advantaged children on non-verbal addition and subtraction problems, and they exhibit few if any differences in the everyday mathematics they employ in free play (Ginsburg, Lee, & Boyd, 2008). Hence, the challenge for teachers is to support less-advantaged children to acquire mathematical language and metacognition – the ability to express and justify their own mathematical thinking – as early in their development as possible.

Children living in less-advantaged circumstances may struggle to participate in mathematics learning contexts that emphasise mathematical discourse as a learning tool (e.g., Lubienski, 2002; Anthony & Walshaw, 2007). Given the key role of language and discourse in mathematics learning, such concerns reinforce the need for intensive instructional support for less advantaged children from an early age (e.g., NRC, 2009) to prepare them to meet the language demands of discourse-based mathematics teaching and learning. Specific strategies include:

- frequent exposure to mathematical language, in both formal and informal contexts (e.g., Klibanoff et al., 2006)
- intentional teaching of mathematical vocabulary using multi-modal methods, with attention to categorisation and associations between related concepts (e.g., Neuman, Newman, & Dwyer, 2011)
- planned opportunities to use language in mathematical problem-solving contexts with varying degrees of structure
- planned opportunities to use mathematical language across a range of curriculum areas (see this volume, Chapter 2, Section: [Practices in Integrative Contexts](#)).

Although there is evidence that children in the urban dimension of the *School Support Programme* (SSP) under DEIS has had some impact on mathematics achievement at second, third and sixth classes between 2007 and 2010 (Weir, Archer, O’Flaherty, & Gilleece 2011), average gain scores are typically small (2–3 standard score points on scales with a mean of 100, and a standard deviation of 15), and it is unclear whether gains are attributable to the SSP as a whole or to one or more of its constituent programmes, such as *Maths Recovery*. The observation that children in DEIS schools continue to lag behind children in non-DEIS schools in mathematics and in other areas of the curriculum (e.g., Eivers et al., 2010) points to a need to intensify the set of mathematics interventions in DEIS schools. While some of the impetus for change will come from the redeveloped curriculum, it is likely that a broader suite of interventions will also be needed. These may include:

- allocation of additional time for mathematics teaching and learning
- ongoing continuous professional development in mathematics for teachers
- affirmation of strategies and programmes that are working effectively to improve children’s mathematics achievement
- access to and support in maintaining and using a broad range of resources for teaching mathematics, including digital learning resources
- intensive learning support interventions for children who are most at-risk that are integrated with classroom instruction
- an emphasis on formative assessment, to complement the strong emphasis on summative assessment in DEIS schools in recent years.

Early Intervention

In a review of the literature on intervention in mathematics, Dowker (2004) makes the case for early intervention, arguing that 'research strongly supports the view that children's arithmetical difficulties are highly susceptible to intervention' (p. 42). In terms of the type of intervention, she emphasises the benefits of individualised instruction:

Moreover, individualized work with children who are falling behind in arithmetic has a significant impact on their performance. The amount of time given to such individualized work does not, in many cases, need to be very large to be effective. (p. 43)

Dowker's (2004, 2009) overviews suggest that targeted interventions based on a diagnostic assessment (see Report No. 17, Chapter 6, Section: [Diagnostic and Summative Assessment](#)) of the strengths and needs of the child in relation to mathematics can be very beneficial and should be a feature of any support system put in place to address low achievement in mathematics. The crucial issue here is not so much the allocation of additional time but rather one of more focused teaching approaches, and clarity about the nature of the learning to be addressed.

There is a key role for the learning support/resource teacher in terms of supporting a prevention and early intervention policy in schools. The *Learning-Support Guidelines* (DES, 2000), while highlighting the role of intensive prevention, interpret early intervention as occurring from senior infants and this needs to be revisited. Many schools only implement early intervention in mathematics from first class onwards (Mullan & Travers, 2010). As noted in Report No. 17, we now have a greater understanding of the range of mathematics that very young children can engage in and the diversity in early mathematical knowledge and skills displayed by children starting school. In addition, we know the importance of disposition and how this can be damaged, affecting engagement and participation in mathematics. This necessitates a much earlier role for prevention and early intervention. The learning support/resource teacher can assist the class teacher in identifying children at risk of mathematical difficulties and engage in in-class as well as external support to address their needs (DES, 2005b).

Allocation of Time to Teaching Mathematics

This section considers the allocation of time to teaching mathematics. First, it looks at allocation of time in preschool settings, and then moves on to primary-school settings. The allocation of time for engagement of children in mathematically-related learning, for example, in working with small groups in a preschool setting, or with larger groups in a primary-school classroom, comprises just one of the contexts in which children may encounter mathematics. Children also engage in mathematics during structured play activities (as recommended in *Aistear*, for example), and in cross-curricular contexts (see below). Throughout this section we emphasise that, while allocation of both dedicated time to mathematics and the integration of mathematics with other areas of learning are important, what is paramount is the quality of the pedagogy (see this volume, [Chapter 1](#), [Chapter 2](#)).

Preschool Settings

Literacy is often seen as an over-riding goal in preschool settings, with considerably less time allocated to mathematics or numeracy (e.g., Lee & Ginsburg, 2009; Thomson, Rowe, Underwood, & Peck, 2005). Now, however, in line with an enhanced understanding of how young children develop mathematically, it is recognised that ‘children require significant amounts of time to develop the foundational mathematical skills and understandings they have the desire and potential to learn and that they will need for success at school’ (NRC, 2009, p. 124). While it is acknowledged that some children can acquire foundational skills at home through spending significant time on focused interaction with family members, it is argued that

Even children who learn mathematical ideas at home will benefit from a consistent high-quality program experience in the preschool and kindergarten years. It is therefore critical that sufficient time is devoted to mathematics instruction in preschool programs so that children develop foundational mathematical skills and understandings... Time must be allocated not only for the more formal parts of mathematics instruction and discussions that occur in the whole group or in small groups, but also for children to elaborate and extend their mathematical thinking by exploring, creating, and playing. (NRC, 2009, p. 124)

An implication of this proposal is that all children should engage in a preschool mathematics programme, in which there is intentional teaching of early mathematics whether in the context of structured whole group or small-group sessions, or in play contexts. Other contexts in which preschool educators can promote mathematical concepts and language include, for example, playing games, reading books with a mathematical theme, using computers, and constructing objects (e.g., block building) (see this volume, Chapter 2, Section: *Practices in Integrative Contexts*). Regardless of the context, however, there is a need for preschool teachers to identify key concepts that children need to learn, and to provide relevant experiences (including materials) that enable children to acquire those concepts. When preschool teachers work with parents to identify opportunities for mathematical development at home, the amount of time in which children attend to mathematical ideas can be increased substantially (NRC, 2009).

Primary School Settings

The PSMC (Government of Ireland, 1999a) suggests that schools allocate a minimum of two hours and fifteen minutes per week to mathematics where there is a short school day for infants, and three hours per week at other class levels. Schools could add discretionary time to this (one hour in the case of infant classes operating with a shorter day, and two hours for other classes), though such time could be allocated to other curricular areas instead.

There have been concerns in Ireland regarding curriculum overload – or the absence of sufficient time to cover all aspects of the curriculum (e.g., NCCA, 2010). One response to such concerns, as

they relate to mathematics education, has been the provision by the NCCA of re-presented content objectives for mathematics (NCCA, 2009c), which seek to present content objectives in a format that renders them more navigable, enabling teachers to more easily see links across objectives between junior infants and second class. A broadly similar approach is used in Northern Ireland, where colour codes are employed to show links across learning statements (CCEA, n.d-a.).

The available evidence suggests that most schools typically exceed the minimum time allocations specified by the NCCA. In the *2009 National Assessments of Mathematics and English* (Eivers et al., 2010), teachers reported allocating 3 hours and 45 minutes to mathematics in second class (and 4 hours and 18 minutes in sixth). Following publication of the *National Strategy to Improve Literacy and Numeracy* (DES, 2011a), where concerns were raised about standards in numeracy (and literacy), the DES issued a circular (0056, 2011), which required schools to increase, from January 2012, the allocation of time spent on mathematics by 70 minutes per week to 3 hours and 25 minutes per week for infants with a shorter day, and to 4 hours and 10 minutes per week for children with a full day. The circular stated that this could be achieved through integrating numeracy with other curriculum areas, using discretionary curriculum time for numeracy activities, reallocating time spent on other subjects in the curriculum to numeracy, and delaying the introduction of other curricular areas. Since the 2009 national assessment of mathematics indicated that teachers of second class were, on average, allocating an average of 3 hours and 45 minutes per week to mathematics, the circular essentially indicates that an additional 25 minutes per week on average (or 5 minutes per day) should be added.

An analysis of data on allocation of teaching time in *Growing Up in Ireland* found that teachers in classrooms of 9-year olds (second to fourth classes) allocated 3.7 hours per week to mathematics in 2007–08 (McCoy, Smyth, & Banks, 2012). While 3 hours or less per week were allocated to mathematics teaching in 40% of primary classrooms, allocation was 5 or more hours in one-quarter of classes. Hence, there is considerable variation around average time allocations. McCoy et al. also reported no significant difference in allocated time for mathematics in DEIS and non-DEIS schools. Male teachers reported spending more time teaching mathematics than females, while teachers in Gaelscoileanna allocated significantly less time to teaching mathematics than their counterparts in other school types. More variation was observed across teachers within schools than across schools in terms of the allocation of time to mathematics teaching, suggesting that teachers enjoyed some autonomy in allocation of instructional time. McCoy et al. also found that the allocation of additional time to mathematics (and English) in the *National Strategy to Improve Literacy and Numeracy* might not affect the time allocated to other subjects if the additional time involved teaching literacy and numeracy across curriculum areas.

Finally, the *2011 Trends in International Mathematics and Science Study* (TIMSS) (Mullis, Martin, Foy, & Arora, 2012), in which children in fourth class in Ireland and in 49 other countries/jurisdictions participated, reported considerable variation in the yearly allocation of time to teaching mathematics. In Ireland, children received 150 hours of mathematics teaching (about 4 hours per week), whereas

Research Report No. 18

Mathematics in Early Childhood and Primary Education (3–8 years)

their counterparts in Northern Ireland, who had a significantly higher average mathematics score than Ireland, received 232 hours (Table 4.1). Ireland's average time was also below the international average of 162 hours. However, TIMSS does not show a linear relationship between time allocation and performance, with, for example, children in Japan also performing well ahead of Ireland, even though they allocated the same amount of annual instructional time. This suggests that allocation of time to mathematics teaching is just one of a number of factors that contribute to actual achievement.

Table 4.1: Annual Allocation of Time to Teaching Mathematics and Mean Achievement – Selected Countries at Grade 4 (TIMSS, 2011)

Country	Annual Time (Hours)	Mean Achievement
Northern Ireland	232	562
Singapore	208	606
United States	206	541
Netherlands	195	540
England	188	542
International Average	162	500
Ireland	150	527
Japan	150	585
Finland	139	545

Source: Mullis et al. (2012), Exhibits 1.1 and 8.6

Of course, allocation of additional time to mathematics instruction may not, by itself, lead to increased mathematical proficiency. According to the NRC report (2001):

[] instruction can best be examined from the perspective of how teachers, students, and content interact in contexts to produce teaching and learning. The effectiveness of mathematics teaching and learning is a function of teachers' knowledge and use of mathematical content, of teachers' attention to and work with students, and of students' engagement in and use of mathematical tasks. Effectiveness depends on enactment, on the mutual and interdependent interaction of the three elements – mathematical content, teacher, students – as instruction unfolds. The quality of instruction depends, for example, on whether teachers select cognitively demanding tasks, plan the lesson by elaborating the mathematics that the students are to learn through those tasks, and allocate sufficient time for the students to engage in and spend time on the tasks. (pp. 8–9)

Allocation of time to mathematics, and, in particular, children's engagement in meaningful mathematical activities, are important factors associated with mathematical proficiency, but, by themselves, they do not guarantee high levels of proficiency, and a range of other factors that contribute to effective instruction need to be considered (e.g., Scheerens, 2004). Nevertheless, sustained time – whether in preschool or school contexts – is an important pre-condition if children are to engage in mathematization and participate in math-talk learning communities.

Emphasis on Different Mathematics Content Areas

In addition to allocation of overall time to mathematics, the allocation of time to individual mathematics content areas may be important. Teachers in TIMSS 2011 in Ireland indicated that 56% of instructional time in mathematics in fourth class was allocated to Number, 22% to Geometry and Measures (combined), 12% to Data Display, and 10% to other topics (Close, 2013). Close argues that the Irish data reflect an over-emphasis on Number, a domain on which children in Ireland tend to do well in national and international assessments. There is a concomitant under-emphasis on Shape and Space, Measures and associated problem-solving activities, on which children in Ireland tend to do less well. While these data relate to fourth class, they suggest a need to ensure a better balance in time allocation across mathematics content areas, with proportionally less time allocated to Number and procedural activities, and more time allocated to Shape and Space and Measures.

In the next section, we look at how mathematics activities might be integrated into other areas of learning, not only to provide children with additional mathematical experiences, but also to help them to see the relevance of mathematical ideas in a broad range of other contexts.

Mathematics Across the Curriculum

In Chapter 2, we identified several practices that teachers can use to support the development of mathematics in different curricular areas and to support children in applying their mathematical knowledge in contexts beyond the mathematics classroom. Here, we consider how proposals to integrate mathematics across the curriculum might impact on curriculum development and implementation, with reference to similar efforts in other jurisdictions.

In discussing the use of mathematics across of range of contexts in early childhood, Perry and Dockett (2002) note that ‘the development of mathematical knowledge and skill go hand-in-hand with their application. Just as mathematics is learned in context, so it is used in context to achieve some worthwhile purpose’ (p. 82). This view is consistent with the ‘integrate and connect’ process skill in the current PSMC (Government of Ireland, 1999a), which includes:

- connecting informally-acquired mathematical ideas with formal mathematical ideas
- recognising mathematics in the environment
- carrying out mathematical activities that involve other areas of the curriculum.

Perry and Dockett (2008) note that ‘the contextual learning and integrated curriculum apparent in many early childhood – particularly prior-to-school – settings ensures that there is little distinction to be drawn between numeracy, mathematical literacy and aspects of mathematical connections with children’s real worlds’ (p. 83).

In Ireland, the *National Strategy to Improve Literacy and Numeracy 2011–2020* (DES, 2011a) emphasises the value of teaching numeracy across the curriculum. The cross-curricular emphasis in the strategy arises from (a) a recognition of the importance of numeracy; and (b) the need to ensure that children have additional opportunities to engage in mathematics, beyond dedicated mathematics time.

In recent years, a number of jurisdictions have begun to emphasise the application of mathematics across the curriculum and across the school day. In Australia, the Australian Curriculum, Assessment and Reporting Authority (ACARA) argues that ‘using mathematics skills across the curriculum both enriches the study of other learning areas, and contributes to the development of a broader and deeper understanding of numeracy’ (n.d., p. 1). It states that, in order to promote numeracy, teachers need to

- identify the specific numeracy demands of their learning area
- provide learning experiences and opportunities that support the application of students’ general mathematical knowledge and skills
- use the language of numeracy in their teaching as appropriate.

To support teachers in doing this, ACARA provides six numeracy learning levels covering foundation year to grade 10. For each level, examples of relevant applications are provided, and specific objectives in English, mathematics, science and history are cross-referenced with examples of cross-curricular mathematical activities.

Using Mathematics is identified as one of three key cross-curricular skills in Northern Ireland (the others are Communication and Using ICT). As well as teaching mathematics as a curriculum area in its own right, teachers are required to teach and encouraged to assess mathematics as a cross-curricular skill. At Key Stage 3, teachers are provided with levels of progress in the cross-curricular skills to support them in assessing children (though the assessments are not statutory). The levels, which are intended to span the primary school years, include reference to

- choosing the appropriate materials, equipment and mathematics to use in a practical situation
- using mathematical knowledge and concepts accurately/working systematically and checking their work
- using mathematics to solve problems and make decisions/developing methods and strategies, including mental mathematics
- identifying and collecting information/reading, interpreting, organising and presenting information in mathematical formats
- using mathematical understanding and language to ask and answer questions, talk about and discuss ideas and explain ways of working
- developing financial capability
- using ICT to solve problems/present their work (CCEA, n.d-b).

In engaging in these activities, children are expected to draw on their knowledge and understanding of number, measures, shape and space and handling data. There is no doubt that this initiative represents a considerable challenge to teachers, both in terms of teaching and assessment, and it is unclear at this time what advantages accrue from making it a focus of assessment.

While the curriculum initiatives in Australia and Northern Ireland designed to promote numeracy across the curriculum are indeed innovative, there is limited evidence to support them, though, as noted in Report No. 17 and in [Chapter 5](#) in this report, some effects have been found for integrating literature and mathematics. If the redeveloped mathematics curriculum for 3- to 8-year-olds promotes children's use of numeracy across learning areas, by, for example, cross-referencing learning outcomes in mathematics with learning outcomes in other curricular areas, it would be important to consider the supports that teachers might need to implement the model, and to evaluate its effects on children's mathematical development. In particular, it would be important to examine how adept teachers are at identifying opportunities to integrate mathematics into other

subject areas, and what supports they might need to do this, particularly in the early stages of integration. It would also be important to ascertain if the allocation of time to the integration of mathematics (as suggested by the *National Strategy to Improve Literacy and Numeracy*) is as effective in promoting children's mathematical understanding as allocating additional time to mathematics as a specific area of learning in its own right.

Conclusion

While the specification of processes and content in the curriculum is critically important, attention must also be given to other issues that support optimal implementation. Among these is access by all children, including exceptional children and children in culturally diverse contexts, to the mathematics curriculum. Other issues relate to the timing of early intervention in mathematics, the allocation of time to mathematics in early learning settings, and the integration of mathematics across the curriculum. If addressed appropriately, and in combination with good mathematics pedagogy, they will contribute towards the realisation of an equitable mathematics curriculum.

The key messages arising from this chapter are as follows:

- The curriculum should recognise the particular challenges of working with exceptional children (including those with intellectual and developmental difficulties, and children with mathematical talent) and with children in culturally diverse contexts.
- A range of approaches and interventions is needed to ensure that children in disadvantaged circumstances reach their full potential in mathematics learning.
- While good mathematics pedagogy meets the needs of all children, an emphasis on mathematical discourse can play a significant role in supporting mathematical learning in the range of language contexts (e.g., English language learners and children in Irish-medium schools).
- A much earlier entry point than is indicated in the *Learning Support Guidelines* is suggested for the provision of early intervention designed to meet the needs of children who are at-risk of experiencing mathematical difficulties. Support for these children should focus on modifications to pedagogy designed to address their needs.
- Sustained time in preschool and primary school contexts – both dedicated and integrated – is an important precondition for children's engagement in mathematics learning.
- It is important that children engage with all of the domains of mathematics, and that opportunities to establish connections between domains are maximised.
- The mathematics curriculum should support an integrative approach across learning areas. One of the ways that this might be achieved is by the provision of exemplars illustrating good practices.

CHAPTER 5

Partnership with Parents



The important role of parents in supporting their children's mathematical development was referred to several times in Report No. 17, in the context of describing approaches to developing mathematical understanding (see Chapter 3, Section: [Adult Support](#)) and assessing mathematics (see Chapter 6, Section: [Supporting Children's Progression with Formative Assessment](#)). In this chapter, we take an in-depth look at how parents can support their children's mathematical development, in partnership with preschools/schools and the wider community. First, we consider how parents can support children's mathematical development in the context of the broader relationship between home and educational settings. Second, we examine effective programmes and partnerships for enhancing children's mathematical learning, and stress the importance of a two-way flow of information. Third, we focus on the role of discussion between parents and children in developing mathematical concepts and mathematical language. Fourth, we look in more detail at initiatives involving parents and teachers, including those implemented in disadvantaged contexts and those on reporting to parents. Fifth, we look at specific activities at home in which parents can engage with their children to develop mathematical understanding. In describing these activities, we emphasise the importance of children's agency in managing their learning.

Parents and Their Children's Mathematical Learning

Epstein (1995) promotes the idea that parental and community involvement should be encouraged to acknowledge the major spheres of influence that affect children's learning: family, school and community. It is generally accepted that the home-learning environment has a powerful effect on children's educational achievement (Anthony & Walshaw, 2007; Epstein, 1995; LeFevre et al., 2009). One of the targets listed in the *National Strategy to Improve Literacy and Numeracy* (DES, 2011a) is to enable parents and communities to support children's numeracy development. However, although decades of research have been conducted on home literacy experiences and strong recommendations for certain parental practices have been made, the same level of research has not been carried out

into children's early mathematical experiences (LeFevre et al., 2009, p. 55). Anthony and Walshaw describe the situation as follows: 'Parents know what it means to *read* with children, yet they are often unclear about what it means to *do mathematics*' (2009b, p. 161, original italics). Maher (2007) found that partnerships between parents and teachers of young children were not as established in mathematics as in reading in a New Zealand-based study. It is likely that this may also be the case in Ireland. Literacy initiatives that target the wider community also appear to be more common than community mathematics initiatives. The Home School Community Liaison (HSCL) Coordinators (2006) describe a number of literacy initiatives where home, school and community work together. One such initiative is the 'One book, one community' scheme, where all members of the community are encouraged to read and discuss the same book (O'Brien Press, 2013). Examples of similar community-based initiatives targeting mathematics are harder to find.

When working with parents to support their children's mathematical learning, it may be necessary to address parents' lack of confidence in their own understandings of mathematics and/or possible alienation (Muir, 2012). Cannon and Ginsburg (2008) note gaps between parents' attitudes to and understandings of mathematics in the early years and highlight the need to educate parents about the mathematics children may learn through daily activities. Changes in approaches to the teaching of mathematics may also make it difficult to involve parents. There is a need to educate parents on the purposes and nature of current approaches, as they might not value such aspects as the incorporation of games or manipulatives (Civil, 2006; Muir, 2012). It is likely that the same holds true for any proposed curriculum change. Bleach (2010) discusses the possibility of 'overload' in Irish primary schools in relation to policy changes and states that it is difficult for schools both to implement changes, and inform and educate parents about those changes. However, she notes that parents cannot effectively support their children if they do not understand the changes being made.

There is a danger that teacher-led practices aimed at involving parents, particularly those in lower socio-economic groups, in their children's education might be based on a deficit model where it is envisaged that input is needed from educators to correct a perceived deficit in the home environment (Edwards & Warin, 1999; Hanafin & Lynch, 2002; Whalley, 2001). However, the literature suggests that, in general, parents of all socio-economic backgrounds wish to support the mathematics education of their children (Anthony & Walshaw, 2007). Parental involvement often involves mothers rather than fathers (Byrne & Smyth, 2010; Whalley, 2001, 2007), and parents in higher socio-economic groups are often more visible in formal involvement with schools as members of parents' associations or boards of management (Bleach, 2010; Hanafin & Lynch, 2002). However, this involvement frequently has limitations and Hanafin and Lynch (2002) report parents' feelings of frustration with these formal structures, where their input is limited to fundraising and 'rubber-stamping' school initiatives rather than shaping them.

At times, there appears to be a mismatch between Irish primary parents' and teachers' perceptions of desirable parental involvement in formal curriculum matters (Mac Giolla Phádraig, 2003a). MacGiolla Phádraig (2003b) notes that, although neither parents nor teachers appear to want to

increase levels of parental involvement at policy level, this stance is at variance with official departmental policy. Disparate opinions on desirable levels of parental involvement in their children's mathematics education may also be linked to value judgements about whose knowledge about mathematics and education is valid, with a higher value often being put on the knowledge of professionals by both parents and teachers alike (Merretens, 1993; Conaty, 2006a).

It seems that partnership approaches should inform the earliest stages of planning interventions and should involve key stakeholders – parents and teachers/practitioners – from the outset (e.g., Whalley, 2001, 2007). From their review of research in this area, Desforges and Abouchaar (2003) suggest that in addition to goodwill and a willingness to work, the following conditions are necessary to increase effective parental involvement:

- strategic planning which embeds parental involvement schemes in whole-school development plans
- sustained support, resourcing and training
- community involvement at all levels of management from initial needs analysis through to monitoring, evaluation and review
- a commitment to a continuous system of evidence-based development and review
- a supportive networked system that promotes objectivity and shared experiences (p. 70).

They also maintain that positive effects on student achievement will result from attention to specific educational goals. In the case of young children learning mathematics, this would suggest that parental involvement activities should target specific mathematical learning goals. The *National Strategy to Improve Literacy and Numeracy* also suggests that engagement with parents should be a core part of schools' literacy and numeracy plans (DES, 2011a), and this is further emphasised in the literature that supports school self-evaluation (DES, 2012).

Communicating with Parents about Mathematics

It is sometimes suggested that a 'communications gap' exists between teachers and parents where teachers' professionalism may act as a barrier to genuine communication (Crozier, 2000). Parents of children attending schools in economically disadvantaged areas have reported feelings of unease when talking with teachers (Hanafin & Lynch, 2002). Parents in socio-economically disadvantaged areas have also been found to be more reluctant to question the teacher or ask for clarification (Hall et al., 2008). Communication issues can be compounded when parents have literacy problems, or additional language issues (Evangelou, Sylva, Edwards, & Smith, 2008) and would appear to be particularly prevalent in the discussion of mathematics, which seems to be less accessible for some parents than other subject areas (Merttens & Newland, 1996). Effective programmes and partnerships often take a holistic approach where sustained mutual collaboration leads to the development of long-term relationships that support positive social change (Anthony & Walshaw,

2009b; Bleach, 2010; Evangelou et al, 2008; Goos, Lowry, & Jolly, 2007; Galvin, Higgins, & Mahony, 2009). This understanding of partnership foregrounds a two-way flow of information.

Sharing Information with Parents

The guidelines for working with Parents outlined in *Aistear* (NCCA, 2009b) recommend the sharing of information with Parents about the curriculum, about children's progress and their learning activities. The guidelines suggest ways of communicating with parents about mathematics, for example, formal meetings about the nature of the curriculum and methodologies, using notice boards, newsletters or photographs to document activities, and providing suggested activities for home. It is also recommended that resources could be shared with parents and that parents could be invited to spend time in the setting. The teacher guidelines accompanying the PSMC also recommend a whole-school sharing of information with parents, particularly in regard to reports of children's progress, and whole-school policies in mathematics and homework (Government of Ireland, 1999b). This contrasts with the less formal approach which has been reported in some early childhood care and education settings. In a study of community childcare centres in the Dublin Docklands, which used the Pen Green approach to parental involvement (Whalley, 2001), 'a quick chat' was often the expected level of parental involvement, for both parents and practitioners (Share, Kerrins, & Greene, 2011, p. 7).

A key practice of the Pen Green Centre is the enhancement of parents' understandings of key child development concepts, sometimes using video-clips of children's activity, so that parents may recognise schemas or patterns of behaviour used by their children (Arnold, 2001). This practice acknowledges the role of parents both as educators and learners. Parents of children attending the Pen Green Centre are offered the opportunity to make recordings of their children engaged in activities at home and discuss it with the centre staff. This could be considered a step forward from transmission-model workshops where curriculum information is simply presented to parents. Instead, this model expects parents to act as educators too and values their observations about their child's activity. Finding ways to support parental understandings of key stages of development in mathematics would go some way towards helping parents support the mathematical development of their children. Parents may benefit from discussions about what constitutes mathematically-rich activity for young children (see Chapter 2, Section: [*Practices in Integrative Contexts*](#)).

A Two-way Flow of Information

Efforts to create a two-way flow of information appeared hard to implement in practice when the Pen Green approach was used in community childcare centres in Dublin's Docklands (Share et al., 2011). This scheme recommended keeping a portfolio of children's work as a starting point for discussion with parents. Epstein (1995) also suggests sending home a portfolio of children's work on a weekly or monthly basis for comments and review as one means of attempting to facilitate two-way communication. Traditional ways of communicating with parents include the use of

parent-teacher meetings and reports of student progress. Mac Giolla Phádraig (2005) suggests that parent-teacher meetings should go beyond a one-way flow of information where teachers provide a report that is generally based on summative assessments of learning (Hall et al., 2008). Instead, these meetings provide an opportunity to agree on priorities for the child's education and for the sharing of information (Mac Giolla Phádraig, 2005). Anthony and Walshaw (2007) suggest that parents may contribute to assessment and Shiel, Cregan, McGough and Archer (2012) note the possibilities for parents to contribute information on their young child's oral language in settings beyond the school or Early Childhood Care and Education (ECCE) setting. Some teachers reported that the mid-year report templates, which were used in an NCCA school-based development initiative, supported effective communication at parent-teacher meetings (NCCA, 2008). Some report templates included the sections 'ways you can help your child' or 'next steps in your child's learning' for explicit advice to parents on how to support their child's learning. Parents reacted positively to the inclusion of these pointers. However, some teachers found it challenging to complete the templates in this manner and were reluctant to give formal advice that might create pressure for parents and children. Since 2011, all schools are required to use NCCA templates to report to parents (DES, 2011b).

The HSCL Scheme is an example of a scheme which aims to increase cooperation in the education of students between schools, parents and community agencies as a means of addressing disadvantage (Ryan, 1994). The goals and principles of the scheme are outlined by Conaty (2006b), who notes the positive possibilities of partnership in terms of its potential for empowerment of individuals and transformation of relationships. A two-way flow of information is envisaged in the 'local committee' element of the HSCL scheme which was described above. At primary level, there is some involvement in classroom activities such as paired reading and targeted programmes such as *Mathematics for Fun* (HSCL Coordinators, 2006). Almost all principals and coordinators reported a positive impact on students, parents and schools (Archer & Shortt, 2003). However, in general, it was perceived that the parents most in need of assistance did not become involved in HSCL projects.

Humphrey and Squires (2011) report on a major intervention, *Achievement for All* (AfA) across 454 schools in England designed to support schools and local authorities to provide better opportunities for learners with special educational needs and disabilities (SEND) to fulfil their potential. AfA had a significant impact on progress in mathematics and English. A key component identified in the success was the structured conversations with parents. These focused on the use of a clear framework for developing an open, on going dialogue with parents about their child's learning. Training was provided for schools, which emphasised the building of parental engagement and confidence via a four-stage model (explore, focus, plan, review) in up to three structured conversations each year with parents in reviewing individual goals.

Technology and Communicating with Parents

Developments in information technology can also create opportunities for new ways of communicating (Desforges & Abouchar, 2003). In fact, the Ballymun Whitehall Area Partnership with Hibernia Consulting (2009) interviewed a number of parents and found that, apart from people they meet or speak to, the internet was the main source of information for how to support their children's learning. Preschools and schools can make more use of technology to communicate with parents. For example, schools and ECCE settings could build on the 'tip sheets' and videos of sample activities for parents available on the NCCA website to provide details on appropriate mathematical learning activities for different age levels. School and ECCE websites could provide information about, and examples of, appropriate mathematical learning activities, resources and links to further information. They could also use websites to detail (with videos, digital photos, samples of student work etc.) the on-going mathematical learning activities of the ECCE setting/classroom and suggest follow-up activities for parents. Also ways to facilitate feedback from parents, either in a face-to-face or online, could be considered. It is possible that parents themselves may use tools such as video or digital cameras to record their child's mathematical activities in the home. Clarke and Robbins (2004) report on a study where parents were provided with a disposable camera and asked to document their preschool children's literacy and mathematical activity. It seems that the task itself increased awareness of the range of mathematical experiences that occur in daily life: parents categorised photos as related to number and other aspects of mathematics. The range of mathematical activities appeared to surprise teachers, and it is likely that variations on this project may be useful for developing discussions between teachers and parents as well as between parents and children.

Parents and Children Discuss Mathematics

Benigno and Ellis (2004) suggest that differences in young children's mathematical abilities may be related to the different kinds of social activities in which they engage in at home. Research suggests that although parents engage in a variety of numerical activities with their children, they do not always utilise opportunities to promote their child's numeracy skills (Benigno & Ellis, 2004; Tudge & Doucet, 2004; Vandermaas-Peeler et al., 2012). The nature of the content of mathematical discussion between parent and child is important and effective parent-child mathematical discussion should move beyond counting to incorporate more complex goals (Skwarchuk, 2008). Skwarchuk found that, while parents often focused on number sense, they were unsure how to incorporate other areas such as measure.

The significance of discussion between parent and child as an influence on student achievement is supported by Desforges & Abouchar (2003). Sheldon & Espstein (2005) suggest that activities where families engage in discussion on mathematics while engaging in mathematics activities may contribute to improving children's mathematical skills. This is further supported by Siraj-Blatchford et al. (2002), who report a positive effect on young children's achievement when parents used

discussion-based learning activities at home. As reported earlier, discussion-based learning is more effective when characterised by sustained shared interactions (see Chapter 2, Sections: [Promotion of Math Talk](#) and [Interactions during Story/Picture-Book Reading](#)).

The nature of parent-child interaction is important, with Pomerantz, Moorman, and Litwack (2007) contending that effective parental interaction supports the development of the child's autonomy, is focused on the process of learning rather than the performance of the child, and is characterised by positive parental affect and positive beliefs about the child's potential. This has salencies with the descriptions of two contrasting parental pedagogical styles described by Aubrey, Bottle and Godfrey (2003), one didactical and one where the focus was on the child's participation. One mother appeared to recognise communication as important, viewed mathematics as part of everyday life and viewed learning opportunities as likely to arise from play. In contrast, another mother took a more direct teaching role and appeared to see mathematics in the home as a series of discrete activities with counting and arithmetical skills as a primary goal. The authors note the potential for a possible disconnect with school, where teaching approaches may sometimes be perceived by parents as more formal than those in ECCE settings. Vandermaas-Peeler, Nelson, Bumpass, and Sassine (2009) recommend that parents should be provided with suitable examples of mathematical discussion that may arise in day-to-day life and that early childhood educators should encourage parents to incorporate mathematics into readings of picture books as well as play activities.

Parents and Teachers Collaborating about Mathematics Learning

There is a number of possibilities for parental involvement in schools, including the use of parents' rooms, curriculum meetings or workshops, and parents assisting on outings or in the classroom (Border & Merttens, 1993). Some of these activities are also recommended in the *Aistear* framework (NCCA, 2009b), in the Home School Liaison Scheme (HSCL Coordinators, 2006) and in various other policy documents (e.g., DES, 1995). Having parents physically present in schools makes visible efforts to encourage parental involvement. However, simply 'getting the parents in' does not guarantee effective practices (Edwards & Warin, 1999; Sheldon & Epstein, 2005). Authors note that subject to how activities are enacted, parents are generally still positioned outside the locus of control of curriculum and pedagogy (Border & Merttens, 1993; Hanafin & Lynch, 2002; Hallgarten, 2000; Mac Giolla Phádraig, 2005), although Anthony and Walshaw (2009b) discuss a number of effective schemes where parents and teachers engaged in genuine collaboration to develop teaching and learning activities.

Muir (2012) describes a 'maths club' which was set up in response to a perceived need to support the parents of students in the senior end of primary school. Attendance rates varied but engagement by parent participants was enthusiastic. The maths club attempted to challenge traditional attitudes to mathematics and activities were based on identified areas of need. The activities gave parents an opportunity to explore how mathematics topics are approached in the

contemporary classroom and provided an opportunity for reviewing preconceptions. Muir notes that the workshops not only address mathematical content but also served a purpose in the affective domain by increasing the confidence of parents and their motivation to do mathematics with their children. This may be particularly important for parents who did not have a positive experience of learning mathematics themselves. It is likely that Muir's approach could be adapted for younger age groups. Civil, Quintos and Bernier (2003) implemented a slightly different approach in the US. While both parents and children were involved in workshops, the children were dismissed during the later stages of the workshops to allow parents to function as adult learners and discuss children's thinking. Such workshops have the possibility to be effective, particularly if parents have some input into the mathematics topics chosen. In both studies, initial workshops were led by outside researchers and the second study used initial outsider input to train local facilitators. This underlines the need for adequate planning and resourcing of parental involvement projects.

Civil et al. (2003) also arranged parent observations of mathematics classes as a means of facilitating dialogue with parents. The dialogue that followed the observations highlighted how parents' own experiences of learning mathematics shaped their perspective of mathematics lessons and what they value as important or 'good' practice. This study focussed on dialogue between researchers and parents rather than parents and teachers. However, it might be possible to adapt the approach to facilitate parent-teacher or parent-practitioner dialogue, possibly using video-recordings for those parents unable to attend during working hours. Such recordings or observations would make it possible for parents to experience approaches to the teaching of mathematics that may be quite different from how they learned the subject (Civil et al., 2003). Having a section of a school or ECCE setting's website dedicated to such video clips could serve as a point of discussion between both parent and child and parent and teacher. Such clips might also provide support to parents and children when engaging follow-up mathematical activities or homework.

It is suggested that HSCL coordinators may function as the 'driving force' behind literacy and numeracy initiatives in schools (HSCL Coordinators, 2006, p. 147) and, as such, it is possible that they may play a role in any new initiatives targeting numeracy activities with parents of children attending DEIS schools. *Mathematics for Fun* is a programme run under the HSCL scheme where parents are invited to participate in and support children's mathematical activities in schools (HSCL Coordinators, 2006). The mathematical activities often focus on the use of manipulatives including tangrams, pattern blocks, dominoes and pentominoes (ibid.). Parents are invited to attend training sessions in the use of the mathematical activities or games and sessions are held over a six-week period, with each session lasting roughly one hour a week. It is suggested that class teachers should be consulted about the suitability of materials but it is unclear how much input the class teacher has into the choice of mathematical activities and it is hard to judge how closely related these activities are to the class scheme for mathematics. Positive feedback from parents, teachers and children about the nature of engagement in activities is reported (HSCL Coordinators, 2006). An evaluation of educational partnerships between Mary Immaculate College, five primary schools, parents, community groups and other organisations lists a wide range of creative partnership projects.

However, numeracy was generally only targeted through the *Mathematics for Fun* programme, related science-focused programmes or through the creation of chess clubs (Galvin, Higgins & Mahony, 2009).

It seems likely that support and training are needed to develop and extend the use of mathematics-focused parental involvement programmes and to broaden the extent of such partnerships to include other members of the community. It also seems likely that such programmes may more directly impact student achievement if they are closely tied to the class learning plan (Desforges & Abouchaar, 2003).

Parent and Child Collaborating about Mathematics

There are a number of mathematical activities that can be used to support young children's learning at home and at school. These include digital and traditional games, number and shape books, number songs and other activities that make use of the environment such as discussions of calendars or money (Vandermaas-Peeler et al., 2012). Anderson, Anderson and Shapiro (2005) examined parent-child exchanges during shared reading sessions with 4-year-old children in their homes. Their findings suggest that, while there was considerable diversity with regard to the type of interactions and the manner in which parents engaged with their children, all but one integrated math talk into the story-reading, especially when discussing the illustrations. The story-related conversations in the home were not in any way contrived and the focus was on the co-construction of meaning. Particular attention centred on concepts such as size and number, as these were seen as arising in a meaningful way within the context of the story.

Homework can be used as a means of facilitating opportunities for parental participation in their children's learning (Merttens & Newland, 1996; Sheldon & Epstein, 2005). Interactive homework may include activities that require parent-child interaction about mathematics or the use of mathematical materials and resources that may be provided by the school (Sheldon & Epstein, 2005). The *Impact* project was a large scale project carried out in the UK which involved parents completing interactive mathematical activities at home with their children (Merttens & Newland, 1996). It aimed to increase opportunities for a two-way flow of information by including opportunities for feedback from parents. Feedback included comments on how enjoyable and accessible the activities were as well as providing opportunities for parents to informally assess their children's mathematical learning. Merttens and Newland (1996) note that having parents assess how their children engaged in the task facilitated more in-depth discussion at later parent-teacher meetings, transforming the experience from 'teacher monologue' to 'dialogue'. They note also that negotiating approaches to a 'school-mathematics' task in the home allows for the regulation and articulation of the task to move between parent and child, particularly when the child acts as instructor to a 'task-naïve' adult (p. 111). This may allow the parent to interact with the child in a way that supports his/her autonomy. This is generally believed to be an effective form of interaction (Pomerantz et al., 2007). Factors affecting the uptake of *Impact* activities included the manner in

which they were introduced to parents, the expectations that were set around acceptable levels of parental involvement and the teacher and his/her role in maintaining contact. The completion of specific activities was sometimes linked to whether parents viewed the activities as 'maths' and whether it related to their understandings of what mathematics education should consist of. This echoes the observations of other authors on parents' perceptions of newer approaches to mathematics (Civil, 2006; Muir, 2012; Pritchard, 2004).

Muir's research on family involvement with 'take home mathematics packs' (2009, 2012) was also based on a similar approach. Muir (2012) describes how activities were designed or selected by the teacher and researcher, based on links to classroom activities, and how a number of 'numeracy bags' were prepared. These bags contained instructions for the activity, necessary materials and guidelines for parents as well as a rationale detailing the mathematical purpose behind the activity. It was intended that children engage in these activities at home with their families two to three times over the course of a week before exchanging the pack for a new numeracy bag. Each numeracy bag also contained a feedback sheet for parents to comment on their child's engagement and any mathematical understandings that were noted. This feedback goes some way to developing communication between parent and teacher.

A strength of both the *Impact* scheme and Muir's scheme is that both focus on numeracy activities that were closely tied to the class learning goals (Desforges & Abrouchaar, 2003). For any practitioner or teacher attempting to initiate such a scheme, attention should be paid to observations of Pomerantz et al. (2007) discussed above. Tasks should be designed and presented in such a way that parents do not feel under pressure to ensure that their children perform in particular manner as this may hinder their inclination to act in process-focused ways that support the autonomy of their children (Pomerantz et al., 2007). Any initiative should be designed with the aim of developing and maintaining the positive affect of parents and positive beliefs about the potential of their children (ibid.)

Conclusion

Parents can become involved in their child's mathematics learning in a variety of ways. This involvement can have positive effects on children's learning. Parental involvement in early education settings should be characterised by a two-way flow of information. Early years educators should highlight with parents the importance of engaging in discussion with their child about mathematically-related activities that arise in the home, and in the context of homework when appropriate. Parents and teachers collaborating and sharing information has been found to be advantageous to teachers, parents and children.

The key messages arising from chapter are as follows:

- There is a need to inform parents about the importance of mathematics learning in the early years, and what constitutes mathematical activity and learning for young children.

- Communication with parents needs to emphasise how the redeveloped curriculum can foster children's engagement with mathematics and the significant role of parents in supporting children's learning.
- Digital technologies offer great potential for communication between parents and educators about young children's mathematical development.
- There is a range of activities in which parents can engage with schools so that both parents and educators better understand children's mathematics learning.

CHAPTER 6

Teacher Preparation and Development



As already elaborated in this and the previous volume, the shift in perspective on what it means for children to learn and use mathematics in the early years demands a change in pedagogy; in particular it puts the teaching-learning relationship at the heart of mathematics (Report No. 18, Chapter 1). This shift requires that educators engage in mathematics teaching in a manner that is qualitatively different from how they themselves learned mathematics (Corcoran, 2008). Putting the teaching-learning relationship at the heart of mathematics acknowledges the equable, or otherwise, outcomes of particular teaching practices and the influence of teacher beliefs and attitudes on these (Bibby, 2009). In the context of defining pedagogy as being about relationship, Bibby (2009, p. 123) contends that '[r]elationships are hard work: They involve knowledge and thinking that goes beyond the rational'. She frames them in terms of 'being in and with' (p. 127) the efforts of learners in the mathematics classroom. This requires that educators be personally committed to teaching mathematics well.

If children are to learn and use mathematics in the coherent and connected manner outlined in Volumes 1 and 2 of this report, educators must acknowledge its importance as more than 'just one of the subjects that I have to teach' (Brown & McNamara, 2005, p. 11). This shift in perspective from traditional teaching of mathematics as rules and procedures, to mathematics teaching as developing mathematical proficiency has important implications for teacher education and sustained teacher development. Among the implications for teacher preparation and development that we have already highlighted as important are:

- a background knowledge of developmental progressions in mathematics learning
- provision for a diverse range of learners
- familiarity with the principles and features of good mathematics pedagogy
- understanding of the role of play in young children's mathematics learning
- incorporation of key meta-practices (math talk, the development of a productive disposition, an emphasis on mathematical modeling, the use of cognitively challenging tasks and formative assessment) in everyday mathematics activity

- a focus on the overall aim of mathematical proficiency and cognisance of the key role that mathematisation plays in progressing this aim.

It is recognised in the literature that teacher preparation and development are complex endeavours that benefit from community support (Krainer, 2005; Jaworski, 2006; DES, 2011a). Below, we discuss key ideas arising from research that should inform professional development.

The Goal of Mathematics Teacher Preparation

We have identified mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) as a key aim of mathematics education. In terms of early years teaching, teachers need a strong working knowledge of mathematics and an openness to and facility for problem-solving. Much of the research in mathematics teacher education of the past twenty-five years has focused on ‘a well organised and flexibly accessible domain-specific knowledge base’ (De Corte, 2004, p. 282). Changes in perspective on what it means to know and use mathematics in teaching can be seen where the research emphasis has moved from the importance of teachers’ subject matter knowledge of mathematics (Ball, 1988) to a widespread acceptance that pedagogical knowledge is the more important teacher variable in student achievement (Education Committee of the European Mathematical Society (EMS), 2012). It follows that the emphasis in mathematics teacher preparation programmes must rest equally on developing both mathematics and mathematics education. The two goals of mathematics teacher preparation must therefore be a) to inculcate a mathematical disposition in future educators, b) together with learning the pedagogic skills and competencies to foster and promote mathematical proficiency in their children (Hiebert, Morris, & Glass, 2003). The design and provision of mathematical experiences that lead to progressive development of each of the strands of mathematical proficiency is primarily the responsibility of the educator in the learning environment. It requires a pedagogy underpinned by principles which relate to people and relationships, the learning environment and the learner (see Chapter 1, Section: [Features of Good Mathematics Pedagogy](#)). To do this effectively, educators need substantial knowledge of mathematics. They need to develop skills for promoting math talk, developing productive dispositions, emphasising mathematical modeling, selecting cognitively challenging tasks and assessing learning (see Chapter 2, Section: [Meta-Practices](#)).

Mathematical Knowledge for Teaching (MKT)

The construct of MKT is claimed to conceptualise the specialised knowledge teachers need in order to complete the tasks of teaching mathematics (Ball & Bass, 2003). MKT is thought to consist of Subject Matter Knowledge and Pedagogical Content Knowledge. Subject Matter Knowledge is further categorised as ‘common content knowledge’ (CCK) and ‘specialised content knowledge’ (SCK). CCK has been identified by recent research in the US as mathematical knowledge present in the population at large (Ball, Thames, & Phelps, 2008) and SCK represents the ‘specialised’

knowledge of mathematics teachers need in order to teach mathematics successfully. Ball and colleagues adapted Shulman's (1986, p. 9) pedagogical content knowledge (PCK) 'for teaching [mathematics]' as Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Content and Curriculum (KCC), all of which they claim can be measured psychometrically.

MKT Research in Ireland

Elements of mathematical knowledge for teaching (MKT) have been enumerated as responding to children's questions, choosing useful examples for highlighting salient mathematical issues, planning lessons, appraising and modifying textbooks and assessing children's learning (Delaney, 2010). These 'tasks' arise out of research of teaching mathematics in the US and Delaney has drawn attention to the mathematical work of teaching in Ireland and the mathematics Irish teachers 'know'. He adapted multiple-choice items developed by Ball, Bass and colleagues for use in Ireland. In these items, teachers are asked, for example, to identify child errors (from a classroom scenario) or to select appropriate representations for particular mathematics problems. The main finding of his study is that mathematical knowledge for teaching varies widely. While his work is based primarily on records of existing practice, Lampert (2001) and Boaler and Staples (2008) propose a relational approach to the development of teacher knowledge – that is, where the educator empathises with the learners as they come to know mathematics. This is discussed further below (see *Effective Teachers' Framework*).

Profound Understanding of Fundamental Mathematics

A small but highly respected study by Ma (1999) demonstrates how powerful mathematics teacher knowledge can be. She offers a comparison of the performance of American and Chinese teachers on four mathematical domains and finds that Chinese teachers demonstrated a consistently stronger conceptual understanding of mathematics than the American teachers and were in all instances better able to explain their 'knowledge packages'. This was despite them having less formal training for teaching mathematics. From her research, Ma describes what she considers essential for teaching: a profound understanding of fundamental mathematics (PUFM). It represents a form of mathematical knowledge that is highly connected, and strongly geared to teaching. In her concluding chapter, Ma argues that what American teachers need is not 'more mathematics' but a 'refocus on teacher preparation' which involves:

Rebuilding a solid and substantial school mathematics for teachers and students to learn... a substantial school mathematics with a more comprehensive understanding of the relationship between fundamental mathematics and new advanced branches of the discipline...indeed, unless such a school mathematics is developed, the mutual reinforcement of low-level content and teaching will not be undone. (p. 149)

From a teacher preparation perspective, it is also worth paying attention to the collaborative and reflective practices among Chinese teachers, which undeniably influenced the development of PUFM. Undoubtedly, educators wishing to progress the aim of mathematical proficiency must have deep and connected knowledge of fundamental mathematics. We also know that effective teaching of mathematics requires considerably more of teachers than being able 'to do' the mathematics they plan to teach; for example, the teaching of subtraction to young children requires more of the teacher than merely knowing how to perform the standard decomposition algorithm (Rowland, 2007). In her research with pre-service teachers, Corcoran (2008) used an audit developed by a team of teacher educators for the SKIMA (Subject Knowledge in Mathematics) project in the UK (Rowland, Martyn, Barber, & Heal, 2001). This audit gives an indication of some of the mathematical strengths and weaknesses of student teachers. She found that while there was a close relationship between mathematics results on a SKIMA audit and mathematics results on the Leaving Certificate Examination, neither was indicative of the quality of mathematics lessons that the student teachers in her study conducted. This indicates that mathematics teaching is a highly complex activity and mathematics teacher preparation requires acknowledgement of the situated, social and distributed nature of mathematics teacher knowledge (Lave, 1988). Mathematics pedagogy then, like mathematical proficiency, is seen as a complex whole, a set of interconnected parts.

There is currently no common mathematics syllabus for pre-service early years or primary teachers. The PSMC and its accompanying *Teacher Guidelines* (Government of Ireland, 1999a; 1999b) indicate the mathematics that primary teachers need to teach and might be interpreted also as indicative of the mathematics that teachers need to know. In fact, teachers of primary mathematics for children aged 3–8 years need to be able to use substantive mathematics, including algebraic thinking, generalisations, equations, functions and graphs, mathematical reasoning and proof, if they are to challenge children to think mathematically (e.g., Ma, 1999).

'Doing' Mathematics

In order to ensure that all children have access to rich mathematics and powerful mathematical ideas, pre-service educators (and practising teachers) need to engage regularly in challenging mathematical activities. By doing interesting and appropriate mathematical investigations and tasks in groups, educators can learn about learning mathematics through 'mathematization' and the opportunities it allows for processes such as communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting. Effective teaching of mathematics requires facility with these processes. They can be developed and their implications for classroom practice can be maximised through explicit mathematical communication and regular, reflective communal engagement with rich mathematics tasks in various contexts. A problem may exist where educators have had previous negative relationships with mathematics and so teacher preparation programmes must respect and model effective mathematics pedagogy by:

- a) acknowledging that all students irrespective of age or previous experience can develop positive mathematical identities and become powerful mathematical learners and teachers
- b) responding to the multiplicity of thinking process and realities found in everyday classrooms with interpersonal respect and sensitivity
- c) focusing on optimising a range of desirable academic outcomes that include mathematical proficiency and a mathematical disposition
- d) committing to enhancing a range of social outcomes within the mathematics classroom that will contribute to the holistic development of participants for productive teaching (adapted from Anthony & Walshaw, 2009b).

By framing mathematics education courses along these lines, teacher educators can provide mathematically-rich learning environments that allow for personal agency and communal support so that pre-service educators may craft positive identities both as learners and teachers of mathematics. By addressing the development of a flexible knowledge base in fundamental mathematics, and engaging thoughtfully in appropriate problem-solving activities, teacher educators can also help their students to become aware of their self-regulation in learning and using mathematics.

Frameworks for Thinking about Pedagogy

The vision of 'good mathematics – taught well' (Even & Lappan, 1994) is integral to the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, 1991, 2000). This is similar to the vision we propose for the redeveloped mathematics curriculum. Both the NRC report (2005) and Anthony and Walshaw's research synthesis (2007) emphasise the importance of frameworks, or systems for thinking about teaching, learning and the design of learning environments. Whichever mathematics curriculum one is teaching, much the same substantive and syntactic mathematics knowledge are required. So what is 'good mathematics' for Irish early years classrooms? And what constitutes 'good mathematics teaching' in Irish terms? The answer may lie in how the curriculum is interpreted and also in how mathematics is understood and valued (Huckstep, 2007; Corcoran, 2008). It is also based on a grounded understanding of learning pathways in mathematics.

The Knowledge Quartet

The Knowledge Quartet (KQ) is a powerful framework devised to aid the development of mathematical knowledge for teaching (Rowland, Huckstep, & Thwaites, 2005). Arising from studying primary mathematics teaching, the Knowledge Quartet identifies four dimensions along which teachers' mathematical knowledge impacts on teaching:

1. *Foundation* is knowledge of the mathematics to be taught, and of theories of teaching and learning mathematics. It includes attitudes and beliefs about mathematics knowledge and pedagogy.
2. *Transformation*, or knowledge-in-action, how to re-present ideas to make them better understood by children, resonates with Shulman's pedagogical content knowledge (1986). Transformation is manifest in a teacher's facility with the art of question-posing and the astute choice of examples.
3. *Connection* involves the ability to sequence material to be taught and an awareness of the relative difficulty for children of different curricular elements. It involves making connections between disparate parts of the lesson or series of lessons and resonates with the 'connectionist' teacher (Askew, 1999).
4. *Contingency*, the more generic ability to deal creatively with the unexpected direction in which a lesson may go, is arguably the most intellectually challenging and difficult to acquire of the four components of the Knowledge Quartet.

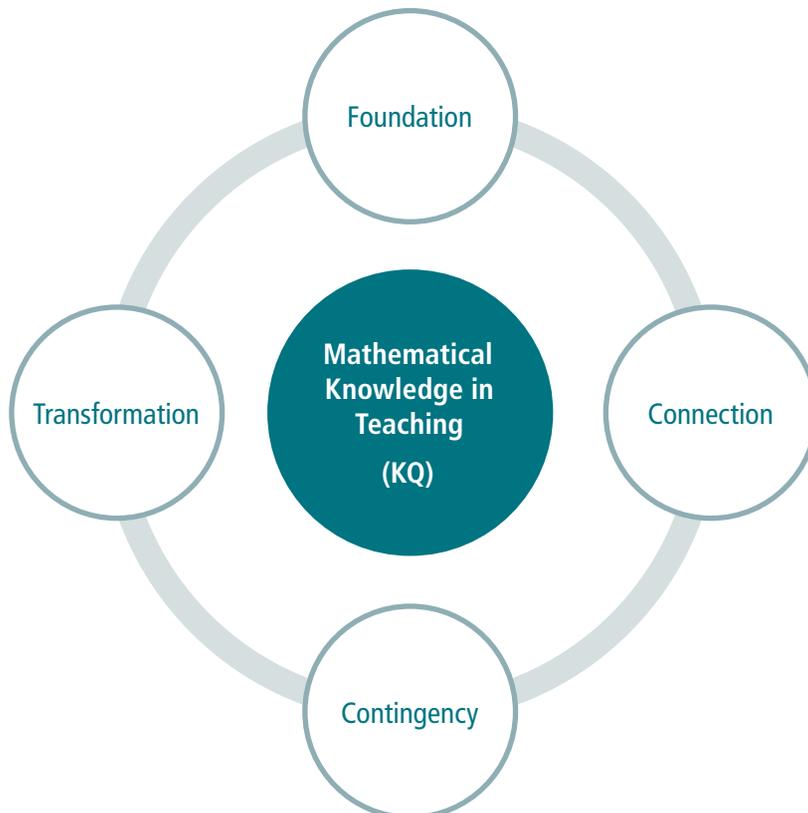


Figure 6.1: The four dimensions of the Knowledge Quartet feed into each other

Each of the four dimensions of the KQ is associated with a certain number of codes or indicators that can be identified with the teacher's activities in planning and teaching a mathematics lesson. The interconnectedness of the four dimensions is evident in the activity of teaching; however, Corcoran (2012) identifies contingency as the most central dimension and of greatest importance in developing mathematical proficiency. Contingency teaching is continually focusing on learners and responding to learners' ideas.

The KQ framework can be used in a number of ways to help in the preparation and delivery of mathematics teaching. It was devised originally to assist tutors who were not mathematics specialists in discussing lessons with student teachers. The twenty contributory codes that feed into the four dimensions of the Knowledge Quartet (Rowland et al., 2005) constitute a 'common technical vocabulary' for talking about teaching (Grossman & McDonald, 2008). Their use is recommended in teacher education as a means of talking about teaching, where they can help build community by becoming part of the 'shared repertoire of ways of doing things' (Wenger, 1998, pp. 82–84). They have been used successfully in a longitudinal study of primary mathematics teacher development in the UK (Turner & Rowland, 2011).

Table 6.1: Contributory codes to the Knowledge Quartet

KNOWLEDGE QUARTET DIMENSIONS	INDICATORS	
FOUNDATION	<ul style="list-style-type: none"> ▪ Adherence to textbook ▪ Awareness of purpose ▪ Concentration on procedures ▪ Identifying errors 	<ul style="list-style-type: none"> ▪ Overt subject knowledge ▪ Theoretical underpinnings of pedagogy ▪ Use of terminology
TRANSFORMATION	<ul style="list-style-type: none"> ▪ Choice of examples ▪ Choice of representation 	<ul style="list-style-type: none"> ▪ Teacher demonstration
CONNECTION	<ul style="list-style-type: none"> ▪ Anticipation of complexity ▪ Decisions about sequencing ▪ Making connections between concepts 	<ul style="list-style-type: none"> ▪ Making connections between procedures ▪ Recognition of conceptual appropriateness
CONTINGENCY	<ul style="list-style-type: none"> ▪ Deviation from agenda ▪ Responding to children's ideas ▪ Use of opportunities 	<ul style="list-style-type: none"> ▪ Teacher insight ▪ Responding to (un)availability of tools and resources

In the Knowledge Quartet framework, isolated practices are not the focus; rather it is the way in which the different elements of the system interact that is important.

Effective Teachers' Framework

Earlier we listed the main features of effective pedagogy by reference to the principles of people and relationships, the learning environment and the learner. Anthony and Walshaw (2009b) elaborate further on what effective teachers do in terms of pedagogy. They start with an 'ethic of care.' This principle underlines the relational aspects of mathematics teaching and learning and the educator's responsibility to be 'in and with' the learners as they 'struggle with mathematics' for themselves, as members of a mathematics learning community. This is followed by the responsibility of educators to 'arrange for learning' by putting learners' current knowledge and interests at the centre of their planning for play, individual, pair, group and whole class work as appropriate. Teachers who 'build on student's thinking' in the design of mathematical tasks are better able to adjust the complexity level of tasks to challenge low-achieving learners. Focus on children's thinking also helps teachers promote the processes of mathematization by knowing when and how to increase the task challenge level. The next two principles are related to classroom discourse. Effective teachers of mathematics facilitate 'mathematical communication' and model the use of appropriate 'mathematical language'. 'Assessment for learning' of mathematics is integral to classroom discourse and effective teachers provide learners with multiple pathways to evaluate and assess their own work. Effective teachers use a variety of 'worthwhile mathematical tasks' and help learners 'make connections' across mathematics, between different solution paths in problem-solving and between mathematics and everyday life. Effective teachers of mathematics carefully choose 'tools and representations' to stimulate and support learners' thinking. Finally, the 'knowledge and learning' of effective mathematics teachers is substantial and robust. It includes 'grounded understanding of students as learners'. In outlining their views of effective mathematics pedagogy, Anthony and Walshaw (2009b) have moved 'away from prescribing pedagogical practice, towards an understanding of pedagogical practice as occasioning students outcomes' (p. 158), with resultant implications for teacher preparation and development.

Using Tools for Teacher Preparation

As research perspectives on mathematics teacher education have moved from an emphasis on teacher knowledge to a more child-focused and community-based approach, elements of Shulman's proposed knowledge base for teaching have been revisited. His proposal was for a) propositional knowledge, b) case knowledge and c) strategic knowledge (Shulman, 1986). According to Shulman:

Case knowledge is knowledge of specific, well-documented, and richly described events. Whereas cases themselves are reports of events or sequences of events, the knowledge they represent is what makes them cases. The cases may be examples of specific instances of practice-detailed descriptions of how an instructional event occurred – complete with particulars of contexts, thoughts, and feelings.

Silver et al. (2007) outline – among other means of learning to teach mathematics effectively—the modus operandi of the COMET (Cases of Mathematics Instruction to Enhance Teaching) project,

which used written case studies in mathematics to effect teacher development. The idea behind the project is that reflection on events in specific classrooms enables teachers to begin to think in a more general way about important matters in mathematics teaching and learning, such as, for example, the need for connections. Similarly, a digitally-based interactive teacher development programme comprising 'records of practice' has been devised between colleagues in the US and representatives of RME in The Netherlands (Fosnot et al., 2013). The re-imagining and re-configuring of the BEd programme in the Irish context constitutes an ideal opportunity to build four-year teacher preparation programmes that include a rich bank of 'cases' of early years mathematics teaching/learning in different mathematical domains. These cases could be based on mathematics learning in a range of education settings in Ireland. Borko, Jacobs, Eiteljor, and Pittman (2008) argue for the effectiveness of videos of children learning mathematics as a tool for teacher education. These new ways of teacher preparation and development emphasise collaboration.

They require pre-service teachers to gain experience with 'approximations of practice' and to focus on a core set of 'high-leverage practices' (Ball, Sleep, Boerst, & Bass, 2009; Grossman, Hammerness, & McDonald, 2009). Thus, strategic knowledge about learning paths in mathematics can be explored and developed before educators begin to teach in 'live' educational settings.

Mathematics Teacher Development (CPD)

International research on mathematics teacher development points to a strong need for mathematics teacher education that continues beyond teacher certification (Krainer, 2011). Teacher participation in professional development as teachers of mathematics has traditionally been very low in Ireland (Delaney, 2005). Except where a small number of teachers have pursued masters or diploma courses in mathematics education, or have been trained to deliver a specialised mathematics programme (e.g., learning support), a five-day summer course is the most likely form of professional development that teachers access. Among the recommendations for in-post teacher development is investment in stronger systems of clinical supervision across the preparation-induction boundary (Grossman, 2010). The notion of clinical supervision could mean an emphasis on developing good mathematics teaching practices through collaborative review and reflection on existing practice. This is important because inquiry as a stance has been advocated as a successful key to teacher change (Jaworski, 2006).

A meta-analysis of teacher professional development research in the US shows that growth in teaching can be achieved through

1. building teachers' mathematical knowledge and their capacity to use it in practice
2. building teachers' capacity to notice, analyse and respond to students' thinking
3. building teachers' productive habits of mind
4. building collegial relationships and structures that support collegial work (Doerr, Goldsmith, & Lewis, 2010).

From their findings it emerges that the key shift involved is one of agency for teachers: from programmes that try to change teachers to teachers as active learners shaping their own professional growth through reflective participation in professional development programmes and in practice. There are numerous examples of such programmes and practice. For example, in Report No. 17 (Chapter 6, Section: *Formative Assessment*) we discussed an Australian project which sought to improve mathematics and numeracy outcomes through working with developmental learning outcomes and a set of powerful mathematical ideas. Lesson study is a practice that is currently foregrounded in the literature as a significant development in school-based professional development.

In lesson study, publicly available records of practice or ‘actionable artifacts’ are important by-products (Lewis et al., 2006, p. 6). It offers opportunities at school and classroom level for enactment of critical inquiry into mathematics lessons (Jaworski, 2006). Noticing children’s responses is an explicit objective and becomes integral to teaching through participation in lesson study. Reflection on mathematics teaching becomes public and is shared through a common language (e.g., that associated with Knowledge Quartet) for talking about teaching. This makes lesson study a particularly effective vehicle for mathematics teacher development (Krainer, 2011) and it is noteworthy that it is gaining support internationally (see, for example, Corcoran & Pepperell, 2011; Fernández, 2005; Hart, Alston, & Murata 2011; Peterson, 2005). Congruent with sociocultural theories of learning, already outlined in Report No. 17, teacher professional development has been found to be more effective when it is sustained, local and supported by the school community (e.g., Cochran-Smith, 2012; Morgan, Ludlow, Kitching, & O’Leary 2010). From this perspective, lesson study is particularly beneficial when enacted by a community of educators working in their own setting, that is, where colleagues are mutually engaged in the shared enterprise of developing mathematical proficiency in their learners. In designing in-service programmes in relation to the redeveloped mathematics curriculum for 3- to 8-year-old children, school-based lesson study should be given due attention.

Conclusion

Teaching-learning relationships are at the heart of mathematics learning. This requires educators to engage in mathematics teaching in a manner that is qualitatively different from how they themselves experienced the learning of mathematics. If pre-service and in-service educators of young children are to promote good mathematics learning, they must have a well-organised and flexibly accessible domain-specific knowledge base. They should have a strong working knowledge of mathematics and an openness to, and facility with, the processes of mathematization. The construct of mathematical knowledge for teaching is important for teacher preparation and development. It can be developed in different ways depending on the level of experience of the teacher. However, critical and collaborative inquiry needs to underpin all efforts to develop teachers’ expertise.

The key messages arising from this chapter are as follows:

- In order to integrate key meta-practices in pedagogy (math talk, productive disposition, modeling, cognitively challenging tasks and assessment), teachers need a profound understanding of mathematics.
- A profound understanding of fundamental mathematics can be developed by educators through a collaborative focus on teaching and learning of mathematics.
- Educators can develop mathematical knowledge for teaching through engaging in rich mathematics tasks.
- The focus of pre-service and in-service teacher education programmes should be on children's engagement in mathematics and their responses to mathematical ideas – valuable contexts are case studies of children learning mathematics and the practice of lesson study.
- Reflective frameworks (e.g., Knowledge Quartet) facilitate critical inquiry and the use of a common language for talking about learning and teaching mathematics.

CHAPTER 7

Key Implications



The purpose of this report is to inform the redevelopment of the mathematics curriculum for children aged 3–8 years. It builds on the research presented in Report No. 17 (definitions, theories, stages of development, and progression). In addressing the issues on teaching and learning, we focused on research related to pedagogy and curriculum. We drew on a broad range of relevant literature and research studies, particularly those published since the introduction of the current Primary School Mathematics Curriculum in 1999. In line with the research request, we focused on features of good pedagogy as they apply to all children, including exceptional children, children in culturally diverse contexts and children in disadvantaged circumstances. We reviewed research related to curriculum design and presentation, and the specification of goals related to processes and content. We have given attention to research on language, integration, time, working with parents, and teacher preparation and development.

The implications for curriculum development presented here are based on a view of curriculum as being multi-faceted. It comprises documentation in which aims, goals and teaching activities are explicated. However, it also involves what happens in classrooms, i.e., what children learn. There needs to be a good fit between these levels. This means that educators need to work together in interrogating the curriculum and negotiating it at a local level.

Our implications are presented in a context in which there is a growing awareness of the extent of mathematical learning in the pre-school years and its significance for later development. Important contextual factors include developments in preschool provision, the increased involvement of parents in their children's education, the multicultural nature of children's learning environments, the ever-growing presence of technology in all aspects of children's lives, concerns about children's mathematical achievements and attitudes, and an economy in which mathematical knowledge is increasingly valued.

The key implications for the redevelopment of the mathematics curriculum arising from this review of research presented in this report are as follows:

- The curriculum should be coherent in terms of aims and goals relating to both processes and content, and pedagogy.

- The processes of mathematization, that is, communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, should be foregrounded in curriculum documentation and should be central to the mathematical experiences of all children.
- The redeveloped mathematics curriculum needs to acknowledge and build on the pedagogical emphases in *Aistear*.
- In order to facilitate transition, educators across early education settings need to communicate about children's mathematical experiences and the features of pedagogy that support children's learning.
- The principles and features of good mathematics pedagogy as they pertain to people and relationships, the learning environment, and the learner, should be emphasised.
- The overarching meta-practices – math talk, productive disposition, modeling, cognitively challenging tasks, and formative assessment – and the ways in which they permeate everyday practices (e.g., story/picture-book reading and project work) should be clearly explicated.
- Educators should be supported in the design and development of rich and challenging mathematical tasks that are appropriate to their children's learning needs.
- The curriculum should exemplify how tools, including digital tools, can enhance mathematics learning.
- Children should engage with all five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance. The strand of Early Mathematical Activities as presented in the current PSMC should be integrated into the five content areas.
- In the curriculum documentation, critical ideas in each content domain need to be explicated. These critical ideas, derived from learning paths, should serve as reference points for planning and assessment. In presenting these ideas, over-specification should be avoided. Learning outcomes arising from these also need to be articulated.
- Narrative descriptors of mathematical development, that is, descriptions of critical ideas, should be developed in class bands, e.g., two years. These critical ideas indicate shifts in children's mathematical reasoning in each of the content domains.
- The principles of equity and access should underpin the redeveloped mathematics curriculum. The nature of support that enables exceptional children, children in culturally diverse contexts and children in disadvantaged circumstances to experience rich and engaging mathematics should be specified.
- Intervention for children at risk of mathematical difficulties should begin at a much earlier point than is specified in current guidelines.

Research Report No. 18

Mathematics in Early Childhood and Primary Education (3–8 years)

- Learning outcomes in mathematics should be cross-referenced with other areas of learning and vice-versa, in order to facilitate integration across the curriculum.
- Time allocated for mathematics should reflect the increased emphases on mathematization and its associated processes.
- Ongoing communication and dialogue with parents and the wider community should focus on the importance of mathematics learning in the early years, the goals of the mathematics curriculum and ways in which children can be supported to achieve these goals.
- Structures should be put in place that encourage and enable the development of mathematical knowledge for pre-service and in-service teachers. Educators need to be informed about goals, learning paths and critical ideas. Records of practice, to be used as a basis for inquiry into children's mathematical learning and thinking, need to be developed.
- Educators need to be given opportunities to interrogate and negotiate the redeveloped curriculum with colleagues as it relates to their setting and context. Time needs to be made available to educators to engage in collaborative practices such as lesson study.
- Given the complexities involved, it is imperative that all educators of children aged 3–8 years develop the knowledge, skills, and dispositions required to teach mathematics well.
- Given the central importance of mathematics learning in early childhood and as a foundation for later development, mathematics should be accorded a high priority, at both policy and school levels, similar to that accorded to literacy.

GLOSSARY

Glossary

Argumentation

'a social phenomenon; when the cooperating individuals try to adjust their intentions and interpretations by verbally presenting the rationale of their actions' (Krummheuer, 1995, p. 229).

Classroom

refers to any group setting for 3- to 8-year-old children (e.g., preschool, family child care, primary school) (NAEYC/NCTM, 2010/2012).

Communicating mathematically

there are a number of ways that children can communicate in mathematics, including oral, visual, digital, textual and symbolic.

Connecting

the notion of 'connections' in mathematics relates both to those that exist (i) within and between different content areas in mathematics (e.g., within number or between number and measurement), (ii) between mathematics learning and learning in other areas and (iii) between mathematics and the context within which a child lives, works or plays (Perry & Dockett, 2008).

Critical transitions

are key developmental understandings related to that concept or domain that are essential for children's understanding of a particular concept or domain.

Curriculum development

the two levels of curriculum development are (a) conceptualisation of plans and the development of resources for teachers and (b) what teachers 'do' to implement these plans in their classrooms (Remillard, 1999).

Exceptional children

children with developmental delays or who are especially talented at mathematics.

Generalising

involves a shift in thinking from specific statements to more general assertions. Children begin to generalise from an early age – for example in learning to use the term 'cup' to refer to all cups (Mason, 2008).

Intentional teaching

the skill of adapting teaching to the content, type of learning experience, and individual child with a clear learning target as a goal (NRC, 2009, p. 226).

Justifying

can be thought of in terms of self-explanation, which is described as ‘inferences concerning ‘how’ and ‘why’ events happen’ (Siegler and Lin, 2010, p. 85).

Knowledge Quartet

a framework devised to aid the development of mathematical knowledge for teaching. Arising from studying primary mathematics teaching, the Knowledge Quartet identifies four dimensions along which teachers’ mathematical knowledge impacts on teaching: foundation, transformation, connection and contingency (Rowland et al, 2005).

Learning outcomes

expected outcomes related to children’s mathematical learning.

Learning paths

sequences that apply in a general sense to children’s development in the different domains of mathematics.

Math talk

children talking about mathematical thinking and engaging in reasoning, argumentation, justification etc.

Mathematical goals

relate to processes and to content. In the literature, the idea of goals is often conflated with the notion of ‘big ideas’.

Mathematical Knowledge for Teaching

the construct of Mathematical Knowledge for Teaching (MKT) is claimed to conceptualise the specialised knowledge teachers need in order to complete the tasks of teaching mathematics. MKT is thought to consist of Subject Matter Knowledge and Pedagogical Content Knowledge (Ball & Bass, 2003).

Mathematical modeling

from the RME perspective, modeling is seen as an organising activity from which a model emerges (Gravemeijer & Stephan, 2002, p. 148).

Mathematical task

an activity, the purpose of which is to focus students’ attention on a particular mathematical idea (Stein et al., 1996, p. 460).

Meta-practices

overarching practices which characterise good mathematics pedagogy (promotion of math talk, development of a positive disposition, emphasis on mathematical modeling, use of cognitively challenging tasks, and formative assessment).

Modeling problems

realistically complex situations where the problem-solver engages in mathematical thinking beyond the usual school experience and where the products to be generated often include complex artefacts or conceptual tools that are needed for some purpose, or to accomplish some goal (English & Sriraman, 2010, p. 173).

Narrative descriptors

descriptors of critical ideas in each content domain. These indicate shifts in mathematical thinking at key transitions.

Number sense

characterised as a holistic concept of quantitative intuition, or a feel for numbers and their interrelationships.

Pedagogy

the deliberate process of cultivating development (Bowman, Donovan & Burns, 2001, p. 182); the practice or the art, the science or the craft of teaching (Siraj-Blatchford et al., 2002, p. 27).

Project

an in-depth study of a particular topic undertaken by small groups of children (Katz & Chard, 2000).

Reasoning

generally associated with logic and the drawing of valid conclusions (e.g., Artzt & Yaloz-Femia, 1999; Steen, 1999).

Records of practice

multimedia case studies or written episodes of classroom vignettes that help educators to situate their own mathematics learning in an authentic classroom experience.

Representing

among the forms of representation that children use to organise and convey their thinking are concrete manipulatives, mental models, symbolic notation, tables, graphs, number lines, stories, and drawings (Langrall et al., 2008). These are sometimes referred to in the literature as 'tools' (see below).

Tools

refer both to physical artefacts and symbolic resources. Physical artefacts include manipulative materials, pens, books and computers, while symbolic resources include language, drawings and diagrams (Armstrong et al., 2005).

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